Event C

Meet 2: Trigonometry

Functions of Sums of angles and sums of functions of angles.
Half and Double angle formulas
Reduction Formulas

$\sin(u+v) = \sin u\cos v + \cos u\sin v$
$\sin(u-v) = \sin u\cos v - \cos u\sin v$
$\cos(u+v) = \cos u\cos v - \sin u\sin v$
$\cos(u-v) = \cos u\cos v + \sin u\sin v$

$\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u\tan v}$

$\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u\tan v}$

$\sin^2 u = \dfrac{1-\cos(2u)}{2}$

$\cos^2 u = \dfrac{1+\cos(2u)}{2}$

$\sin\left(\frac{\pi}{2} - u\right) = \cos u$, $\cos\left(\frac{\pi}{2} - u\right) = \sin u$

$\tan\left(\frac{\pi}{2} - u\right) = \cot u$, $\cot\left(\frac{\pi}{2} - u\right) = \tan u$

$\sec\left(\frac{\pi}{2} - u\right) = \csc u$, $\csc\left(\frac{\pi}{2} - u\right) = \sec u$

$\sin(\pi + u) = -\sin u$, $\sin(\pi - u) = \sin u$

$\sin(2u) = 2\sin u\cos u$

$\cos(2u) = \cos^2 u - \sin^2 u$
$= 2\cos^2 u - 1$
$= 1 - 2\sin^2 u$

$\tan(2u) = \dfrac{2\tan u}{1 - \tan^2 u}$

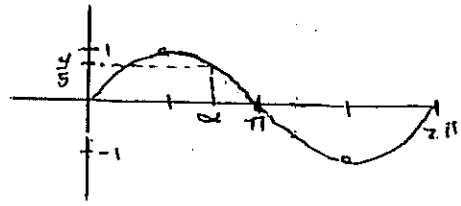
1. If $\sin\alpha = t$, express $\cos(2\alpha)$ in terms of t.

2. If $\tan\alpha = t$ and $\tan\beta = \frac{1}{t}$, find $\tan(\alpha - \beta)$, in terms of t.

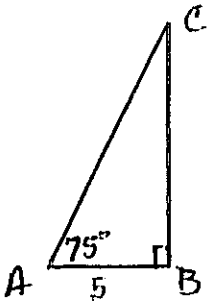
3. Simplify:

$$\sin x + \sin\left(x + \tfrac{2\pi}{3}\right) + \sin\left(x + \tfrac{4\pi}{3}\right)$$

4. The graph of $y = \sin x$ indicates $\sin\alpha = \frac{4}{5}$. What is $\sin(2\alpha)$?



5. Refer to $\triangle BCA$. Find BC, in simplified form.

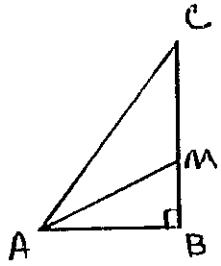


6. Let $\theta = \frac{3\pi}{8}$. Find $\sin\theta + \cos\theta$, expressing your answer in the form $\frac{\sqrt{m+n\sqrt{2}}}{k}$ where m, n, k are integers.

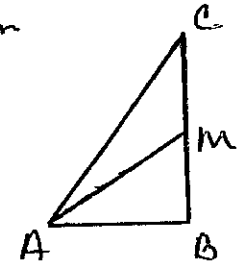
Hint: Begin with $(\sin\theta+\cos\theta)^2$.

7. Refer to the 3-4-5 right $\triangle ABC$ with $AB=3$ and $BC=4$.

a) If $\overline{AM}$ bisects $\angle A$, find $\cos\angle MAC$.



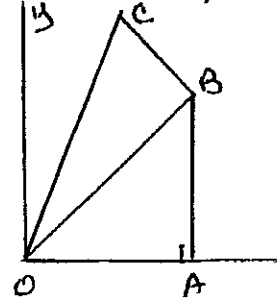
b) If $\overline{AM}$ is the median drawn from A, find $\cos(MAC)$.



8. Solve for x. $0 \le x \le 90°$

$\sin x = 8\sin 10° \sin 50° \sin 70° \sin 80°$

9. In the figure, $\overline{OB}$ is the hypotenuse of isosceles right $\triangle OAB$ and $\overline{OB}$ is the long side of $30°-60°-90°$ $\triangle OBC$ having rt. angle B. Find the coordinates of C, given $OB=2$.



Answers

1. $1-2t^2$ 2. $\frac{t^2-1}{2t}$ 3. 0 4. $-\frac{24}{25}$ 5. $5(2+\sqrt{3})$ 6. $\frac{\sqrt{4+2\sqrt{2}}}{2}$ 7. a) $\frac{2\sqrt{5}}{5}$ b) $\frac{17\sqrt{13}}{65}$ 8. $80°$ 9. $(\sqrt{2}-\frac{\sqrt{6}}{3}, \frac{\sqrt{2}+\sqrt{6}}{3})$