Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have an extended 20 minutes for this event.

_______________ 1. To build a robot, Isaac had to buy some nuts and bolts. There were two different sales going on: he could either get a 15% discount on his entire purchase, or a 50% discount on just bolts (but not both). It turned out that his final cost was the same regardless of which discount he chose to apply. If he originally needed $7 (pre-sale value) worth of nuts, what value (pre-sale) of bolts did he need?

_______________ 2. Kyla has a small stack of crisp $2 bills, and another small stack of $5 bills, that she has collected from various birthday cards. When out to a restaurant for this year's birthday, she realizes that if she uses all of her $2 bills, she will be $60 short toward buying 4 burgers; if she uses all of her $5 bills, she will be $60 short toward buying 5 burgers, and if she uses all of her $2 *and* $5 bills (all of her money), she will be $60 short toward buying 6 burgers. What is the cost of one burger?

_______________ *min.* 3. A small stream is flowing into a pond at a constant rate. A pack of 12 elephants can empty the pond in 4 minutes, while a pack of 9 elephants would do so in 6 minutes. How long would it take 6 elephants to drink the pond dry?

_______________ *trees* 4. There are 18 oak trees in a forest. Each tree has the same number of acorns. A tornado passes through the forest, causing some of the trees to lose their acorns. Some trees lost exactly half their acorns, others lost exactly a third of their acorns, while still others lost none at all. Overall, one-ninth of the total acorns were lost. How many trees lost none?

Name: ______________________________ Team: ______________________________

Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event A

SOLUTIONS

$3

Graders: A dollar sign is not necessary for credit.

1. To build a robot, Isaac had to buy some nuts and bolts. There were two different sales going on: he could either get a 15% discount on his entire purchase, or a 50% discount on just bolts (but not both). It turned out that his final cost was the same regardless of which discount he chose to apply. If he originally needed $7 (pre-sale value) worth of nuts, what value (pre-sale) of bolts did he need?

If b = value of bolts, then $0.85(7+b)=7+0.5b \Rightarrow 5.95+0.85b=7+0.5b \Rightarrow 0.35b=1.05 \Rightarrow b=3.$

$20

Graders: A dollar sign is not necessary for credit.

2. Kyla has a small stack of crisp $2 bills, and another small stack of $5 bills, that she has collected from various birthday cards. When out to a restaurant for this year's birthday, she realizes that if she uses all of her $2 bills, she will be $60 short toward buying 4 burgers; if she uses all of her $5 bills, she will be $60 short toward buying 5 burgers, and if she uses all of her $2 *and* $5 bills (all of her money), she will be $60 short toward buying 6 burgers. What is the cost of one burger?

Let T = # of $2 bills, F = # of $5 bills, and B = cost of a burger. Then we have the system:

$$\begin{cases} 2T+60=4B \\ 5F+60=5B \\ 2T+5F+60=6B \end{cases}$$

Subtracting the second equation from the third yields B = 2T, and substituting that into the first equation, $B+60=4B \Rightarrow B=20$.

12 *min.*

3. A small stream is flowing into a pond at a constant rate. A pack of 12 elephants can empty the pond in 4 minutes, while a pack of 9 elephants would do so in 6 minutes. How long would it take 6 elephants to drink the pond dry?

Let S = rate of stream, E = rate of 1 elephant. Then $S+12E=\frac{1}{4}$ *pond/minute, while* $S+9E=\frac{1}{6}$ *pond/minute. Subtracting the second equation from the first yields* $3E=\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$*, so* $E=\frac{1}{36}$. *By substitution,* $S+12\left(\frac{1}{36}\right)=\frac{1}{4} \Rightarrow S=\frac{1}{4}-\frac{1}{3}=-\frac{1}{12}$*, and so* $S+6E=-\frac{1}{12}+6\left(\frac{1}{36}\right)=\frac{1}{12}$ *pond/minute.*

13 *trees*

4. There are 18 oak trees in a forest. Each tree has the same number of acorns. A tornado passes through the forest, causing some of the trees to lose their acorns. Some trees lost exactly half their acorns, others lost exactly a third of their acorns, while still others lost none at all. Overall, one-ninth of the total acorns were lost. How many trees lost none?

Let n = # of trees that lost no acorns, h = # of trees that lost half. Then 18 – h – n = # of trees that lost one-third. If A = # of acorns per tree, then we have $h\cdot\frac{A}{2}+(18-h-n)\cdot\frac{A}{3}=18\cdot\frac{A}{9}$. *Dividing through by A (the # of acorns per tree is irrelevant!),* $\frac{h}{2}+\frac{18-h-n}{3}=2 \Rightarrow \frac{h}{2}+6-\frac{h}{3}-\frac{n}{3}=2$

$\Rightarrow \frac{h}{6}+6=\frac{n}{3}+2 \Rightarrow h+36=2n+12 \Rightarrow n=\frac{h}{2}+12$. *h* > 0 *and h* + *n* < 18, *so* (*h*, *n*) = (2, 13).

Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

__________ %

1. All 4 side lengths of a square are decreased by 30%. By what percentage does the square's <u>area</u> decrease?

Volume = __________

2. A cube with surface area 72 is sliced into 8 identical smaller cubes. Determine exactly the volume of one of these smaller cubes.

____

3. At the *Grand Ole Creamery* ice cream shop in St. Paul, employees drop a 0.5" diameter malted milk ball into each waffle cone to prevent melted ice cream from leaking out the bottom of the cone. If the cones are 6" tall, with a diameter of 2.5" at the top (see *Figure 3*), calculate how far above the bottom of the cone the center of the malted milk ball will be situated.

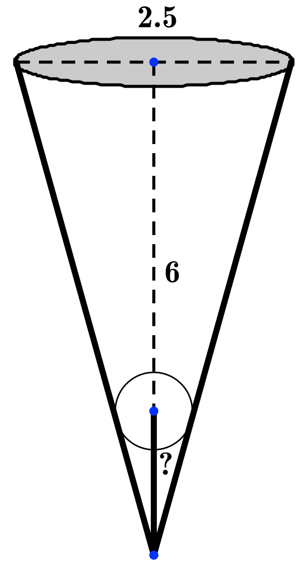

Figure 3

Perimeter = __________

4. In $\triangle ABC$ *(Figure 4)*, point D is on $\overline{AB}$ such that $\angle B \cong \angle ACD$. If $AD = 40$, $BD = 10$, and $CD = 30$, determine exactly the perimeter of $\triangle ABC$.

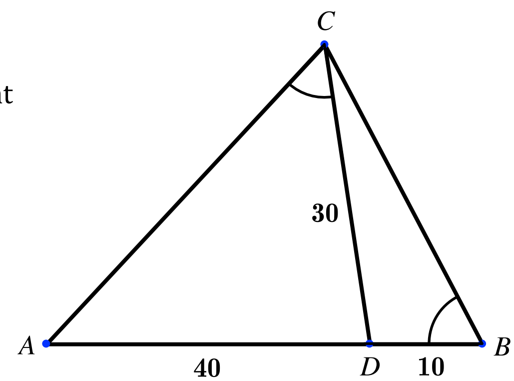

Figure 4

Name: ____________________ Team: ____________________

Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event B

SOLUTIONS

Answer: $\boxed{51}$ %

1. All 4 side lengths of a square are decreased by 30%.
By what percentage does the square's <u>area</u> decrease?

Let x = the original side length. Then the original area is x^2*, and the new area is* $(0.7x)^2 = 0.49x^2$. *This is 49% of the original area, so the area has decreased by* 51%.

Answer: *Volume =* $\boxed{3\sqrt{3}}$

2. A cube with surface area 72 is sliced into 8 identical smaller cubes.
Determine exactly the volume of one of these smaller cubes.

Each of the 6 faces of the original cube will have area 72 ÷ 6 = 12, *so each edge length of the original cube will be* $\sqrt{12} = 2\sqrt{3}$. *The new cubes' edge lengths will be half that (see Figure 2), so their volumes will each be* $\left(\sqrt{3}\right)^3 = 3\sqrt{3}$.

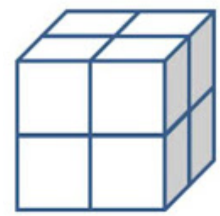

Figure 2

Answer: $\boxed{\dfrac{\sqrt{601}}{20}}$ ***or*** $\boxed{\approx 1.225}$ ***or*** $\boxed{\approx 1.226}$

3. At the *Grand Ole Creamery* ice cream shop in St. Paul, employees drop a 0.5" diameter malted milk ball into each waffle cone to prevent melted ice cream from leaking out the bottom of the cone. If the cones are 6" tall, with a diameter of 2.5" at the top (see *Figure 3*), calculate how far above the bottom of the cone the center of the malted milk ball will be situated.

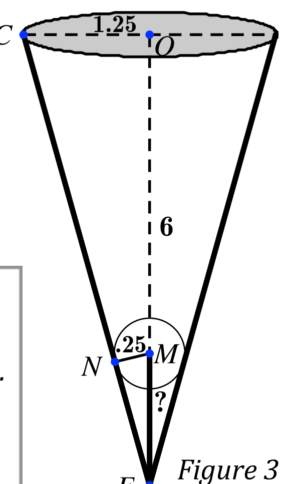

Figure 3

Label Figure 3 as shown. Draw malted milk ball radius $\overline{MN}$*, which is perpendicular to* $\overline{CE}$. $\triangle MNE \sim \triangle COE$*, so* $\dfrac{MN}{NE} = \dfrac{CO}{OE} \Leftrightarrow \dfrac{.25}{NE} = \dfrac{1.25}{6} \Rightarrow NE = \dfrac{6}{5}$.

By the Pyth. Thm., $\left(\dfrac{1}{4}\right)^2 + \left(\dfrac{6}{5}\right)^2 = ME^2 \Rightarrow ME = \sqrt{\dfrac{1}{16} + \dfrac{36}{25}} = \sqrt{\dfrac{601}{400}} = \dfrac{\sqrt{601}}{20}$.

Answer: *Perimeter =* $\boxed{50 + 35\sqrt{5}}$

4. In $\triangle ABC$ *(Figure 4)*, point *D* is on $\overline{AB}$ such that $\angle B \cong \angle ACD$. If *AD* = 40, *BD* = 10, and *CD* = 30, determine exactly the perimeter of $\triangle ABC$.

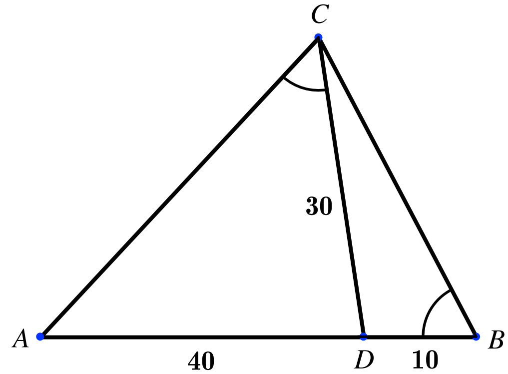

Figure 4

Since $\angle A \cong \angle A$, $\triangle ACD \sim \triangle ABC$ *by AA. So* $\dfrac{40}{AC} = \dfrac{AC}{50}$

$\Rightarrow AC^2 = 2000 \Rightarrow AC = 20\sqrt{5}$. *Furthermore,*

$\dfrac{40}{AC} = \dfrac{30}{BC} \Rightarrow 40 \cdot BC = 30 \cdot AC \Rightarrow BC = \dfrac{30}{40} \cdot 20\sqrt{5} = 15\sqrt{5}$.

Perimeter$(\triangle ABC) = 50 + 20\sqrt{5} + 15\sqrt{5} = 50 + 35\sqrt{5}$.

Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

__________ 1. What is the probability that a fair coin, flipped three times, will land all tails?

__________ 2. Determine exactly the probability that a student, by randomly guessing, achieves a score of 3 out of 5 on a pop quiz whose questions are multiple choice with four choices each.

__________ 3. In how many ways can 9 identical candy bars be distributed to 4 children? *(The candy bars are only distributed in whole units, without being cut.)*

$a =$ __________ 4. Given that $(1+x)^6(1+ax)^7 = 1+bx+15x^2+\ldots+a^7x^{13}$ and $a \neq 0$, determine exactly the values of a and b.

$b =$ __________

Name: ____________________ Team: ____________________

Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event C

SOLUTIONS

$\frac{1}{8}$ *or* 0.125 *or* 12.5%

1. What is the probability that a fair coin, flipped three times, will land all tails?

The probability of flipping tails with a single coin is $\frac{1}{2}$. *Each coin flip is independent of the other two; therefore, the probability of flipping three tails is* $\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$.

$\frac{45}{512}$

2. Determine exactly the probability that a student, by randomly guessing, achieves a score of 3 out of 5 on a pop quiz whose questions are multiple choice with four choices each.

The number of ways a student can get exactly 3 out of 5 questions correct is ${}_5C_3=10$. *The probability of each of these ways is* $\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2=\frac{9}{1024}$, so the desired probability is $10\cdot\frac{9}{1024}=\frac{90}{1024}=\frac{45}{512}$.

220

3. In how many ways can 9 identical candy bars be distributed to 4 children?
(The candy bars are only distributed in whole units, without being cut.)

Using asterisks to represent the candy bars, we need to position 3 dividing lines amongst the asterisks in order to subdivide the candy bars into 4 groups, some of which may be empty:

| * * | | * * * * * * *   *or*   * | * * * * | * * | * *   *or*   * * * | * * * | * * * |

We have 12 total objects, and must choose 3 positions for the dividing lines: ${}_{12}C_3=220$.

$a=$ -2

$b=$ -8

Graders: Award one point per correct value.

4. Given that $(1+x)^6(1+ax)^7=1+bx+15x^2+\ldots+a^7x^{13}$ and $a\neq 0$, determine exactly the values of a and b.

Using binomial expansion on the left-hand side, we obtain
$\left(1+({}_6C_1)1^5x^1+({}_6C_2)1^4x^2+\ldots+x^6\right)\left(1+({}_7C_1)1^6(ax)^1+({}_7C_2)1^5(ax)^2+\ldots+(ax)^7\right)$. *Equating the coefficients of the x terms, we get the equation* $7a+6=b$. *Equating the coefficients of the* x^2 *terms, we get the equation* $21a^2+42a+15=15\Rightarrow 21a^2+42a=0\Rightarrow a=0$ *or* $a=-2$. *Because* $a\neq 0$, $a=-2$ *and* $b=7a+6=7(-2)+6=-8$.

Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

________________ 1. Leyla had just enough money to buy 44 gerbils when she arrived at the store. But a sale was on; each gerbil purchased at full price would allow a second gerbil to be purchased at 40% off. Given the sale, what is the maximum total number of gerbils Leyla can now buy?

________________ 2. For <u>how many</u> positive integers n will $\frac{2n}{18-n}$ also be a positive integer?

________________ 3. Seven consecutive integers, the least of which has a value of a, have an average of b. In terms of a, what is the average of the seven consecutive integers, the least of which is b?

(a, b) = ________________ 4. A line with slope m passing through the point (10, 75) will <u>not</u> intersect $y = x^2$ if and only if $a < m < b$ for some real numbers a and b. Determine exactly the ordered pair (a, b).

Name: ________________________ Team: ________________________

Minnesota State High School Mathematics League

2014-15 Meet 5, Individual Event D

SOLUTIONS

NO CALCULATORS are allowed on this event.

54

1. Leyla had just enough money to buy 44 gerbils when she arrived at the store. But a sale was on; each gerbil purchased at full price would allow a second gerbil to be purchased at 40% off. Given the sale, what is the maximum total number of gerbils Leyla can now buy?

[2014 AMC 12A, problem #2]

The new average price per gerbil will be $1.6 \div 2 = 0.8$ *of the original price. Dividing,* $44 \div 0.8 = 44 \cdot \frac{5}{4} = 55$*, but the 55th gerbil would cost* <u>*more*</u> *than average, so Leyla can only buy* 54.

7

2. For <u>how many</u> positive integers n will $\frac{2n}{18-n}$ also be a positive integer?

[2014 AMC 12B, problem #7]

Substituting the values of 1 through 17 for n, we obtain the following positive integers:
$\frac{12}{12}=1$, $\frac{18}{9}=2$, $\frac{24}{6}=4$, $\frac{28}{4}=7$, $\frac{30}{3}=10$, $\frac{32}{2}=16$, $\frac{34}{1}=34$. *We count 7 valid values for n.*

$\boxed{a+6}$

3. Seven consecutive integers, the least of which has a value of a, have an average of b. In terms of a, what is the average of the seven consecutive integers, the least of which is b?

[2014 AMC 12A, problem #9]

$$\frac{a+(a+1)+(a+2)+\ldots+(a+6)}{7}=\frac{7a+21}{7}=a+3=b \Rightarrow \frac{b+(b+1)+(b+2)+\ldots+(b+6)}{7}=\frac{7b+21}{7}=b+3=a+6$$

$(a, b) = \boxed{(10, 30)}$

4. A line with slope m passing through the point (10, 75) will <u>not</u> intersect $y = x^2$ if and only if $a < m < b$ for some real numbers a and b. Determine exactly the ordered pair (a, b).

[2014 AMC 12B, problem #17]

Using point-slope form, the line's equation is $y-75=m(x-10) \Rightarrow y=mx-10m+75$. *Substituting* x^2*, we have* $x^2=mx-10m+75 \Rightarrow x^2-mx+(10m-75)=0$. *The line and parabola will not intersect iff this quadratic equation has no real solutions, so its discriminant must be negative:* $m^2-4(10m-75)<0 \Rightarrow m^2-40m+300=(m-10)(m-30)<0$. *The left-hand side of this inequality is only negative when* $10 < m < 30$.

Minnesota State High School Mathematics League

2014-15 Meet 5, Team Event

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

_____________ *tokens*

1. A subway station sells 1-ride, 5-ride, and 20-ride cards. Each card costs a whole number of subway tokens. It is cheaper to buy one 5-ride card than five 1-ride cards; it is also cheaper to buy one 20-ride card than four 5-ride cards. A group of 33 friends has pooled together their 33 subway tokens to purchase cards allowing 35 rides, and that was the cheapest way in which they could have bought at least 33 rides.
How many subway tokens did a 5-ride card cost?

Area[ABCD] =

____

2. Three pairs of parallel line segments intersect as shown in *Figure 2*. All of the acute angles in the figure measure 60°, and $AB = CE = 4$.
If trapezoids *ABCD* and *BGFA* are similar, determine exactly the area of *ABCD*.

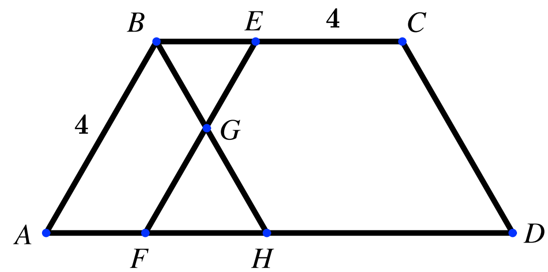

Figure 2

____

3. 4 women each store a distinct hat in the same box. If all 4 women reach into the box randomly and independently, what is the probability that no woman picks her own hat?

_____________ *bees*

4. Some bees are perched on a bale of hay shaped like a rectangular prism. When a bee is perched on an edge, it counts as being "on" both faces intersecting that edge, and when perched on a vertex, it counts as being "on" all 3 faces intersecting that vertex.
If there are a different number of bees "on" each face, and no two bees can perch at the same point, what is the <u>least</u> possible total number of bees on the hay bale?

____

5. Eight congruent isosceles triangles are constructed with their bases on the sides of a regular octagon of side length 2. The sum of the areas of the triangles is equal to the area of the octagon. Calculate the length of an altitude of one of the triangles, drawn from its vertex angle to its base.

____

6. How many 3-digit numbers N have the property that the absolute value of the difference between N and the number formed by reversing the digits of N is 594?

Team: _______________________

Minnesota State High School Mathematics League

2014-15 Meet 5, Team Event

SOLUTIONS (page 1)

$\boxed{5}$ *tokens*

1. A subway station sells 1-ride, 5-ride, and 20-ride cards. Each card costs a whole number of subway tokens. It is cheaper to buy one 5-ride card than five 1-ride cards; it is also cheaper to buy one 20-ride card than four 5-ride cards. A group of 33 friends has pooled together their 33 subway tokens to purchase cards allowing 35 rides, and that was the cheapest way in which they could have bought at least 33 rides.
How many subway tokens did a 5-ride card cost?

Area[ABCD] = $\boxed{8\sqrt{6}+4\sqrt{3}}$

2. Three pairs of parallel line segments intersect as shown in *Figure 2*. All of the acute angles in the figure measure 60°, and *AB* = *CE* = 4. If trapezoids *ABCD* and *BGFA* are similar, determine exactly the area of *ABCD*.

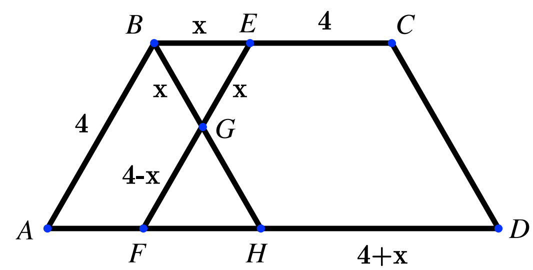

Figure 2

$\boxed{\frac{3}{8}}$ ***or*** $\boxed{0.375}$ ***or*** $\boxed{37.5\%}$

3. 4 women each store a distinct hat in the same box. If all 4 women reach into the box randomly and independently, what is the probability that no woman picks her own hat?

$\boxed{6}$ *bees*

4. Some bees are perched on a bale of hay shaped like a rectangular prism. When a bee is perched on an edge, it counts as being "on" both faces intersecting that edge, and when perched on a vertex, it counts as being "on" all 3 faces intersecting that vertex.
If there are a different number of bees "on" each face, and no two bees can perch at the same point, what is the <u>least</u> possible total number of bees on the hay bale?

$\boxed{\sqrt{3+2\sqrt{2}}}$ ***or*** $\boxed{1+\sqrt{2}}$ ***or*** $\boxed{\approx 2.414}$

5. Eight congruent isosceles triangles are constructed with their bases on the sides of a regular octagon of side length 2. The sum of the areas of the triangles is equal to the area of the octagon. Calculate the length of an altitude of one of the triangles, drawn from its vertex angle to its base.

[2014 AMC 12A, problem #10]

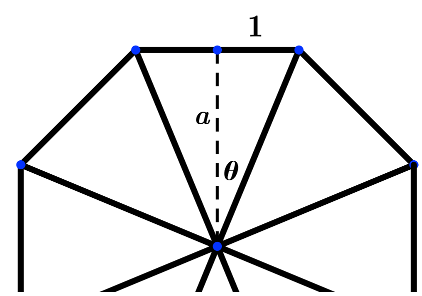

Figure 5

$\boxed{60}$

6. How many 3-digit numbers *N* have the property that the absolute value of the difference between *N* and the number formed by reversing the digits of *N* is 594?

[2014 AMC 12A, problem #6]

Minnesota State High School Mathematics League

2014-15 Meet 5, Team Event

SOLUTIONS (page 2)

1. *Since cards with more rides are always cheaper, the 35 rides must be distributed as one 20-ride card and three 5-ride cards. Let T = cost of 20-ride card; F = cost of 5-ride card. We have* $\begin{cases} T+3F=33 \\ T<4F \end{cases}$ *, with an intersection at* $(T,F)=\left(4\tfrac{6}{7},18\tfrac{6}{7}\right)$. *The graph of this system suggests that we should begin investigating ordered pairs (T, F) where T ≥ 5 and F ≤ 11:* (6, 9), (9, 8), (12, 7), *and* (15, 6) *all have the property that two 20-ride cards would be cheaper; only* (18, $\boxed{5}$) *works*.

2. *Let BE = BG = EG = x. Then FG = 4 − x, and BC = DH = 4 + x. By the similarity of the trapezoids,* $\frac{BG}{AB}=\frac{FG}{BC} \Rightarrow \frac{x}{4}=\frac{4-x}{x+4}$, *so* $x^2+4x=16-4x \Rightarrow x^2+8x-16=0 \Rightarrow x=4\sqrt{2}-4$. *The height of ABCD is the same as that of equilateral triangle ABH,* $2\sqrt{3}$. *Area[ABCD] =* $\frac{1}{2}(BC+AD)(2\sqrt{3})=((4+x)+(8+x))\sqrt{3}=(12+2(4\sqrt{2}-4))\sqrt{3}=(8\sqrt{2}+4)\sqrt{3}=\boxed{8\sqrt{6}+4\sqrt{3}}$.

3. *There are 4! = 24 possible outcomes. Certainly these outcomes could be listed and checked; we will use inclusion/exclusion: For at least one woman to get her own hat, we must choose the one woman, and permute the other 3 hats:* ${}_4C_1\cdot 3!$ *For at least 2 women to get their own hats, we must choose the two women, and permute the other 2 hats:* ${}_4C_2\cdot 2!$ *Continue in this way:* $4!-({}_4C_1\cdot 3!)+({}_4C_2\cdot 2!)-({}_4C_3\cdot 1!)+({}_4C_4\cdot 0!)=24-24+12-4+1=9$, *so P =* $\frac{9}{24}=\boxed{\frac{3}{8}}$.

4. *Let the hay bale's vertices be CUBEC'U'B'E'. Place a bee on vertices C', U', and E', and also on edges CC', U'B', and C'E', as shown in Figure 4. Then there will be 0 bees on the bottom face, 1 bee on the right face, 2 bees on the rear face, 3 bees on the left face, 4 bees on the front face, and 5 bees on the top face, but only a total of* $\boxed{6}$ *bees are on the hay bale. (To see that 5 bees or less are impossible, note that having 0, 1, 2, 3, 4, and 5 bees on the 6 faces is a minimum, so we must have at least 5 bees. Then, referring to the figure, the bee on CC' would need to be removed, and no rearrangement of the 5 bees on the top face works.)*

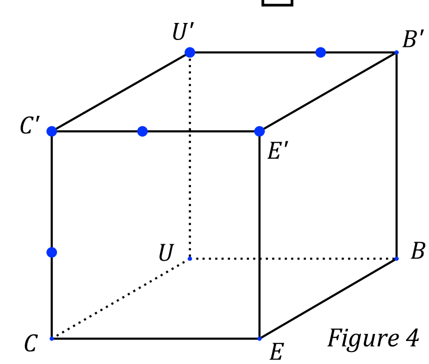

Figure 4

5. *The conditions of the problem can be completely satisfied by simply sectioning the interior of the octagon into its 8 congruent isosceles triangle "wedges"! (see Figure 5) This means that the altitude we're looking for is the same as the octagon's apothem. The apothem forms a right triangle with leg length 1 and central angle* $\theta=22.5°$, *so we have*

$$\frac{1}{a}=\tan\theta=\tan\frac{45°}{2}=\sqrt{\frac{1-\cos 45°}{1+\cos 45°}}=\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}} \Rightarrow a=\sqrt{\frac{2+\sqrt{2}}{2-\sqrt{2}}}=\sqrt{\frac{(2+\sqrt{2})(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})}}=\sqrt{\frac{6+4\sqrt{2}}{4-2}}=\sqrt{3+2\sqrt{2}}=\boxed{1+\sqrt{2}}.$$

6. *Let N =* $\underline{X}\,\underline{Y}\,\underline{Z}$ *= 100X + 10Y + Z. Then the number formed by reversing the digits of N is* $\underline{Z}\,\underline{Y}\,\underline{X}$ *= 100Z + 10Y + X, and the absolute value of the difference of these two numbers is* $|(100X+10Y+Z)-(100Z+10Y+X)|=|99X-99Z|=99|X-Z|$. *Because this equals* 594, *we have* $|X-Z|=6$, *so we are looking for all possible pairs of digits (X, Z) that are* 6 *apart. (Note that neither digit can be 0, because then either N or its reverse would have a leading zero.) The* 6 *possible pairs are* (1, 7), (7, 1), (2, 8), (8, 2), (3, 9), *and* (9, 3), *each of which allows* 10 *choices for Y. This makes* (6)(10) = $\boxed{60}$ *choices for N.*