# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points.
Place your answer to each question on the line provided. You have 12 minutes for this event.

***NO CALCULATORS are allowed on this event.***

______________ 1. Determine exactly the value of $\left(4^{\frac{5}{4}}\right)^{\frac{2}{5}}$.

$(b, c) =$ ______________ 2. Given $\dfrac{x^3-8}{x-2}+\dfrac{3x^3-9x^2+6x}{x^2-3x+2}=x^2+bx+c$, determine exactly the values of *b* and *c.*

*width =* ______________ 3. A particular rectangle has the property that if we triple the width, then add 12, and then take the square root of the result, we obtain the rectangle's length.
Determine exactly the width for which the rectangle will be a square.

$f(x) =$ ______________ 4. If $f\left(\dfrac{1}{x+3}\right)=\dfrac{1}{2-5x}$ for all x > 1, write $f(x)$ as a rational function with no common factors shared by the numerator and denominator.

Name: ______________________________ Team: ______________________________

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event A

## SOLUTIONS

***NO CALCULATORS are allowed on this event.***

$\boxed{2}$

1. Determine exactly the value of $\left(4^{\frac{5}{4}}\right)^{\frac{2}{5}}$.

$$\boxed{\left(4^{\frac{5}{4}}\right)^{\frac{2}{5}} = 4^{\frac{5}{4}\cdot\frac{2}{5}} = 4^{\frac{2}{4}} = 4^{\frac{1}{2}} = \sqrt{4} = 2.}$$

$(b, c) = \boxed{(5, 4)}$

2. Given $\dfrac{x^3-8}{x-2}+\dfrac{3x^3-9x^2+6x}{x^2-3x+2} = x^2+bx+c$, determine exactly the values of *b* and *c.*

$$\boxed{\frac{x^3-8}{x-2}+\frac{3x^3-9x^2+6x}{x^2-3x+2} = \frac{\cancel{(x-2)}\left(x^2+2x+4\right)}{\cancel{(x-2)}}+\frac{3x\cancel{(x-1)}\cancel{(x-2)}}{\cancel{(x-1)}\cancel{(x-2)}} = x^2+5x+4, \text{ so } (b, c) = (5, 4).}$$

*width* = $\boxed{\dfrac{3+\sqrt{57}}{2}}$

3. A particular rectangle has the property that if we triple the width, then add 12, and then take the square root of the result, we obtain the rectangle's length.
Determine exactly the width for which the rectangle will be a square.

*For a rectangle to be a square, the length and the width must be equal. If we let x be the width, then the length is* $\sqrt{3x+12}$. *Setting these expressions equal, we obtain the equation* $x = \sqrt{3x+12}$. *Squaring both sides gives the quadratic equation*

$x^2 = 3x+12 \;\Rightarrow\; x^2-3x-12=0 \;\Rightarrow\; x = \dfrac{3\pm\sqrt{57}}{2}$. *Since the width is positive,* $x = \dfrac{3+\sqrt{57}}{2}$.

$f(x) = \boxed{\dfrac{x}{17x-5}}$

***Also accept:***

$\boxed{\dfrac{5}{17(17x-5)}+\dfrac{1}{17}}$

***or other equivalent expressions that satisfy the given conditions.***

4. If $f\left(\dfrac{1}{x+3}\right) = \dfrac{1}{2-5x}$ for all x > 1, write $f(x)$ as a rational function with no common factors shared by the numerator and denominator.

*We can solve this problem with a change of variable. Let* $z = \dfrac{1}{x+3}$. *Rewriting and solving for x yields* $x = \dfrac{1}{z}-3$. *So* $f\left(\dfrac{1}{x+3}\right) = f(z) = \dfrac{1}{2-5\left(\dfrac{1-3z}{z}\right)} = \dfrac{z}{17z-5}$, *and* $f(x) = \dfrac{x}{17x-5}$.

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points.
Place your answer to each question on the line provided. You have 12 minutes for this event.

$VT =$ ____________

1. In *Figure 1*, $\overline{MT} \perp \overline{RW}$, $VN = VM = VW = 3$, $RN = 1$, and $m\overset{\frown}{MW} = 75°$. Calculate the length $VT$.

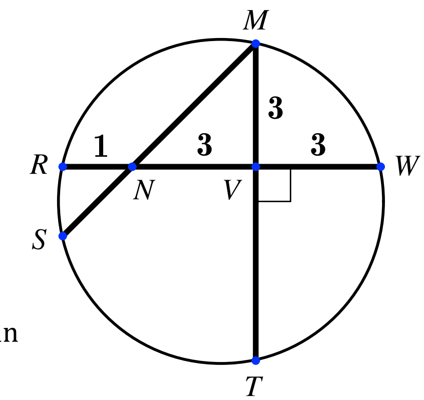


*Figure 1*

$m\overset{\frown}{RS} =$ ____________

2. Again using *Figure 1*, with the same conditions as given in problem #1, find the measure of $\overset{\frown}{RS}$.

$m\angle CEB =$ ____________

3. In *Figure 3*, $\triangle ABC$ is an isosceles triangle inscribed in a circle, with $AB = AC$. If $\overline{CD}$ is a diameter of the circle, and $m\angle CAB = a$, write an expression for $m\angle CEB$ in terms of $a$.

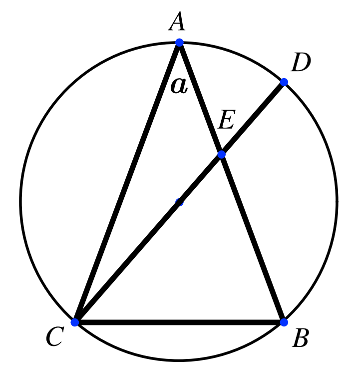


*Figure 3*

Perimeter = ____________

4. Triangle $FGH$ is formed by the pairwise intersections of three tangents to a circle, as shown in *Figure 4*. Two of the tangents ($\overline{FK}$ and $\overline{FL}$) are fixed in position, while the third tangent ($\overline{GH}$) intersects the circle at a movable point ($P$). If $FK = 27$, calculate the perimeter of $\triangle FGH$.

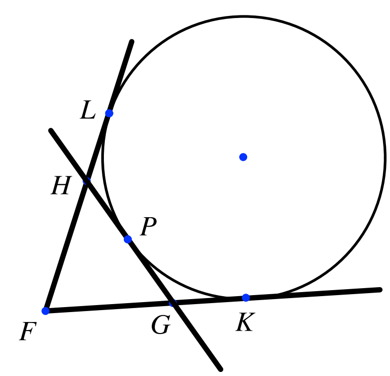


*Figure 4*

Name: ______________________ Team: ______________________

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event B

## SOLUTIONS

$VT = \boxed{4}$

1. In *Figure 1*, $\overline{MT} \perp \overline{RW}$, $VN = VM = VW = 3$, $RN = 1$, and $m\overarc{MW} = 75°$. Calculate the length $VT$.

> *Because $\overline{MT}$ and $\overline{RW}$ are two intersecting chords,*
> $RV \cdot VW = MV \cdot VT \;\Rightarrow\; 4 \cdot 3 = 3 \cdot VT \;\Rightarrow\; VT = 4$.

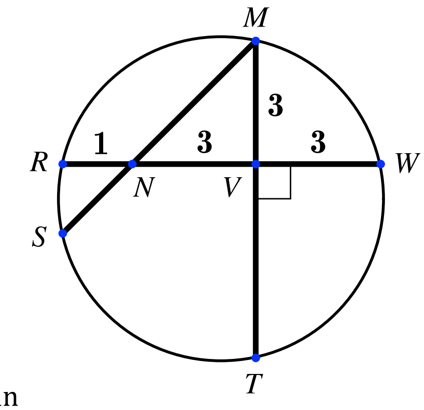


*Figure 1*

$m\overarc{RS} = \boxed{15°}$

2. Again using *Figure 1*, with the same conditions as given in problem #1, find the measure of $\overarc{RS}$.

> *Intersecting chords $\overline{RW}$ and $\overline{MS}$ cut off arcs $\overarc{RS}$ and $\overarc{MW}$, with the relationship that $m\angle MNV = \frac{1}{2}\left(m\overarc{MW} + m\overarc{RS}\right)$. $\triangle MNV$ is an isosceles right triangle, so $m\angle MNV = 45°$, and*
> $m\overarc{MW} + m\overarc{RS} = 2 \cdot 45° \;\Rightarrow\; 75° + m\overarc{RS} = 90° \;\Rightarrow\; m\overarc{RS} = 15°$.

$m\angle CEB = \boxed{\frac{3}{2}a}$

3. In *Figure 3*, $\triangle ABC$ is an isosceles triangle inscribed in a circle, with $AB = AC$. If $\overline{CD}$ is a diameter of the circle, and $m\angle CAB = a$, write an expression for $m\angle CEB$ in terms of $a$.

> *Chase some angles:* $\angle A = a \;\Rightarrow\; m\overarc{BC} = 2a$. *Using semicircle CBD,*
> $m\overarc{BD} = 180° - 2a$, *so* $m\angle DCB = 90° - a$. *Base angle B measures*
> $\dfrac{180° - a}{2} = 90° - \dfrac{a}{2}$, *so* $m\angle CEB = 180° - (90° - a) - \left(90° - \frac{a}{2}\right) = \frac{3}{2}a$.

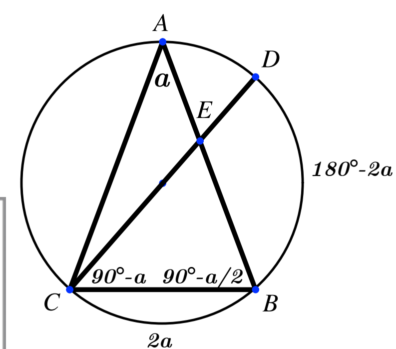


*Figure 3*

Perimeter = $\boxed{54}$

**(Note that if P is movable, we can imagine it sliding along the circle to point K ... making a 2-sided "flat" triangle where both sides are aligned with FK!)**

4. Triangle $FGH$ is formed by the pairwise intersections of three tangents to a circle, as shown in *Figure 4*. Two of the tangents ($\overline{FK}$ and $\overline{FL}$) are fixed in position, while the third tangent ($\overline{GH}$) intersects the circle at a movable point ($P$). If $FK = 27$, calculate the perimeter of $\triangle FGH$.

> *Perimeter$(\triangle FGH)$ = FG + FH + GH = FG + FH + (HP + GP). Since tangents to a circle from the same point are congruent, HP = HL and GP = GK. Thus, we can rewrite the perimeter as FG + FH + (HL + GK) = (FG + GK) + (FH + HL) = FK + FL = 27 + 27 = 54.*

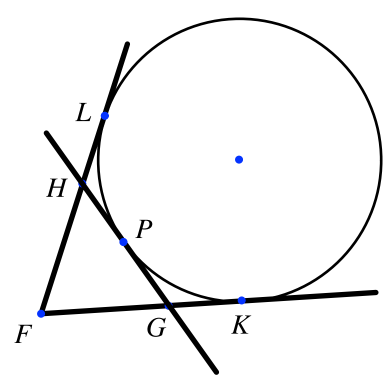


*Figure 4*

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

***NO CALCULATORS are allowed on this event.***

____________ 1. What is the 54th term of the sequence 3, 5, 7, 9, . . . ?

____________ 2. What is the coefficient of the $x^2$ term in $(2x-3)^5$ ?

____________ 3. Determine exactly the infinite sum $8-4\sqrt{2}+4-2\sqrt{2}+2-\ldots$

$f(8)=$ ____________ 4. Given a function where $f(n+2)=2f(n+1)+f(n)$, with $f(1)=x$, $f(2)=2$, $f(3)=x+4$, and $f(4)=x$, determine exactly the value of $f(8)$.

Name: ____________________ Team: ____________________

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event C

## SOLUTIONS

***NO CALCULATORS are allowed on this event.***

$\boxed{109}$ 1. What is the 54th term of the sequence 3, 5, 7, 9, . . . ?

*The explicit formula for an arithmetic sequence is* $a_n = a_1 + d(n-1)$.
*Here, the common difference is two; therefore,* $a_{54} = 3 + 2(54-1) = 109$.

$\boxed{-1080}$ 2. What is the coefficient of the $x^2$ term in $(2x-3)^5$?

*By the Binomial Theorem, the $x^2$ term will be* ${}_5C_3(2x)^2(-3)^3 = 10 \cdot 4x^2(-27)$,
*so the coefficient is* $10 \cdot 4 \cdot (-27) = -1080$.

$\boxed{16-8\sqrt{2}}$ 3. Determine exactly the infinite sum $8 - 4\sqrt{2} + 4 - 2\sqrt{2} + 2 - \ldots$

*This is a geometric series with common ratio* $\dfrac{-4\sqrt{2}}{8} = \dfrac{-\sqrt{2}}{2}$. *Its infinite sum is given by*
$\dfrac{a_1}{1-r} = \dfrac{8}{1+\dfrac{\sqrt{2}}{2}}$. *Multiplying the numerator and denominator of this expression by 2 yields*
$\dfrac{16}{2+\sqrt{2}}$, *and rationalizing using* $2-\sqrt{2}$, *we have* $\dfrac{16\left(2-\sqrt{2}\right)}{\left(2+\sqrt{2}\right)\left(2-\sqrt{2}\right)} = \dfrac{16\left(2-\sqrt{2}\right)}{4-2} = 16 - 8\sqrt{2}$.

$f(8) = \boxed{-362}$ 4. Given a function where $f(n+2) = 2f(n+1) + f(n)$, with $f(1) = x$, $f(2) = 2$, $f(3) = x+4$, and $f(4) = x$, determine exactly the value of $f(8)$.

*Because* $f(4) = 2 \cdot f(3) + f(2)$, *we have* $x = 2(x+4) + 2$. *Solving, we find that* $x = -10$.
*So* $f(1) = -10$, $f(2) = 2$, $f(3) = -6$, *and* $f(4) = -10$, *and we can continue the sequence:*
$f(5) = 2(-10) + -6 = -26$, $f(6) = 2(-26) + -10 = -62$, $f(7) = 2(-62) + -26 = -150$, *and*
$f(8) = 2(-150) + -62 = -362$.

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

***NO CALCULATORS are allowed on this event.***

____________ 1. What is the minimum possible $y$-coordinate on the parabola $y = x^2 - 6x + 10$?

$r =$ ____________ 2. Determine exactly the length of the radius of the circle $x^2 + 6x + y^2 - 4y = 3$.

$c =$ ____________ 3. For what value of $c$ is the ellipse $4x^2 + 4x + y^2 - 6y = c$ tangent to the $y$-axis?

*y-int* = ____________ 4. An ellipse with foci located at (0, 1) and (4, 3) passes through the origin. Determine exactly the ellipse's <u>other</u> $y$-intercept.

Name: ____________________ Team: ____________________

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Individual Event D

## SOLUTIONS

***NO CALCULATORS are allowed on this event.***

$\boxed{1}$ 1. What is the minimum possible $y$-coordinate on the parabola $y = x^2 - 6x + 10$?

*The minimum y-coordinate occurs at the vertex of this upward-facing parabola. Find the vertex either by rewriting in vertex form* $\left(y = x^2 - 6x + 10 = (x-3)^2 - 9 + 10 = (x-3)^2 + 1\right)$, *or by calculating* $-\frac{b}{2a}\left(= -\frac{-6}{2} = 3\right)$ *and substituting* $\left(y = (3)^2 - 6(3) + 10 = 1\right)$.

$r = \boxed{4}$ 2. Determine exactly the length of the radius of the circle $x^2 + 6x + y^2 - 4y = 3$.

*Complete the square on both x and y:*
$\left(x^2 + 6x + 9\right) + \left(y^2 - 4y + 4\right) = 3 + 9 + 4 \Leftrightarrow (x+3)^2 + (y-2)^2 = 16$, *so* $r = \sqrt{16} = 4$.

$c = \boxed{-9}$ 3. For what value of $c$ is the ellipse $4x^2 + 4x + y^2 - 6y = c$ tangent to the $y$-axis?

$$4x^2 + 4x + y^2 - 6y = c \Leftrightarrow 4\left(x^2 + x\right) + \left(y^2 - 6y\right) = c \Leftrightarrow 4\left(\left(x + \frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right) + (y-3)^2 - 9 = c$$

$\frac{\left(x+\frac{1}{2}\right)^2}{1/4} + \frac{(y-3)^2}{1} = c + 10$, *which is an ellipse centered at* $(-0.5, 3)$. *The y-axis is* 0.5 *units from the center, matching the* $\left(1/2\right)^2$ *in the first denominator, so simply set* $c + 10 = 1 \Rightarrow c = -9$.

$y\text{-int} = \boxed{3}$ ***or*** $\boxed{(0,3)}$ 4. An ellipse with foci located at (0, 1) and (4, 3) passes through the origin. Determine exactly the ellipse's <u>other</u> $y$-intercept.

*Even though this is an oblique ellipse, this problem can solved without any special equation. All that is required is the definition of an ellipse as "the locus of points, the <u>sum</u> of whose distances from two fixed points (foci) is a constant." The distances from* (0, 0) *to the foci are* 1 *and* 5 *respectively, so this ellipse's constant sum is* 6. *This means*

$$d_1 + d_2 = (y-1) + \sqrt{4^2 + (3-y)^2} = 6 \Rightarrow \sqrt{25 - 6y + y^2} = 7 - y.$$

*Squaring both sides and solving for y:*

$$25 - 6y + \cancel{y^2} = 49 - 14y + \cancel{y^2} \Rightarrow 8y = 24 \Rightarrow y = 3.$$

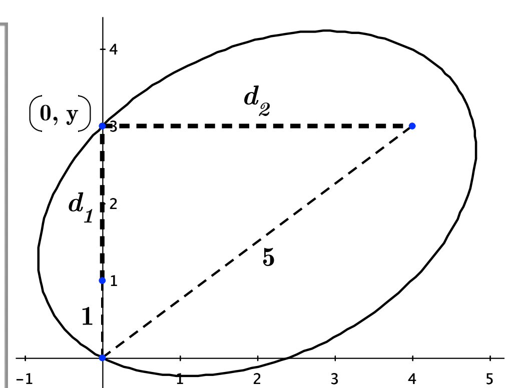


*Figure 4*

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Team Event

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

$f(4) =$ __________

1. Given a function where $f(n+2) = 2 \cdot f(n+1) + 3 \cdot f(n)$, with $f(1) = 5$ and $f(5) = 265$, determine exactly the value of $f(4)$.

$r =$ __________

2. Two perpendicular secants from a point $P$ meet a circle at points $W$, $X$, $Y$, and $Z$, as shown in *Figure 5*. If $PW = 2$, $WX = 4$, and $PY = 3$, calculate the length of the circle's radius.

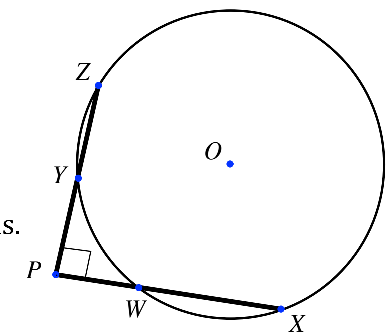


*Figure 2*

*Perimeter =* __________

3. *Figure 3* shows a sequence of figures comprised of adjacent hexagons. The first figure (upper left) shows 2 hexagons. In the second figure (to the right), 1 hexagon is attached, then 2 more are attached, then 1, and so on. If the side length of each hexagon is 1, determine the outer perimeter of the 2015th figure. *(The first perimeter is 10.)*

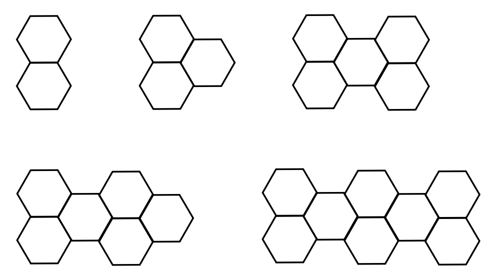

*Figure 3*

__________

4. Two circles, different in size, intersect at points $A$ and $B$. On the larger circle, $m\overset{\frown}{AB} = 40°$, while on the smaller circle, $m\overset{\frown}{AB} = 170°$. If the radius of the larger circle is 14, calculate the radius of the smaller circle.

$c =$ __________

5. Determine exactly the <u>minimum</u> possible value of $c$ such that if $f(x)$ is a straight line passing through (1, 7), then $f(f(x))$ passes through (7, $c$).

__________

6. A circle with center $C$ is internally tangent to $x^2 + y^2 = 16$ and externally tangent to $x^2 - 2x + y^2 = 0$. The collection of all possible points $C$ form an ellipse. Determine exactly the length of the ellipse's minor axis.

Team: ____________________

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Team Event

## SOLUTIONS (page 1)

$f(4) =$ $\boxed{86}$

1. Given a function where $f(n+2) = 2 \cdot f(n+1) + 3 \cdot f(n)$, with $f(1) = 5$ and $f(5) = 265$, determine exactly the value of $f(4)$.

$r = \boxed{\dfrac{\sqrt{65}}{2}}$ **or** $\boxed{\approx 4.031}$

2. Two perpendicular secants from a point $P$ meet a circle at points $W$, $X$, $Y$, and $Z$, as shown in *Figure 5*. If $PW = 2$, $WX = 4$, and $PY = 3$, calculate the length of the circle's radius.

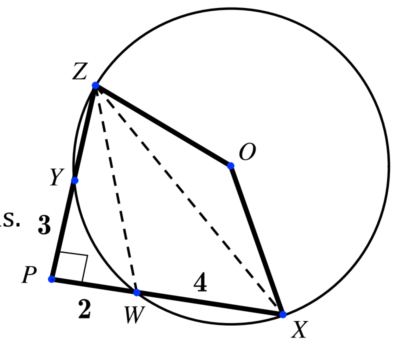


*Figure 2*

*Perimeter* = $\boxed{8066}$

3. *Figure 3* shows a sequence of figures comprised of adjacent hexagons. The first figure (upper left) shows 2 hexagons. In the second figure (to the right), 1 hexagon is attached, then 2 more are attached, then 1, and so on. If the side length of each hexagon is 1, determine the outer perimeter of the 2015$^{th}$ figure. *(The first perimeter is 10.)*

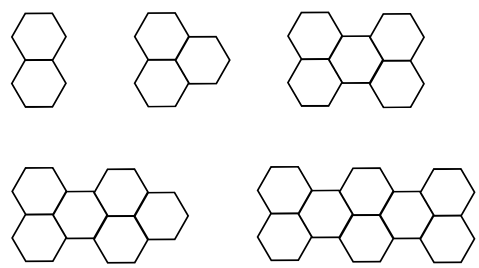

*Figure 3*

$\boxed{\dfrac{14 \cdot \sin 20°}{\sin 85°}}$ **or** $\boxed{\dfrac{14 \cdot \cos 70°}{\cos 5°}}$ **or** $\boxed{\approx 4.806}$ **or** $\boxed{\approx 4.807}$

4. Two circles, different in size, intersect at points $A$ and $B$.
On the larger circle, $m\overset{\frown}{AB} = 40°$, while on the smaller circle, $m\overset{\frown}{AB} = 170°$.
If the radius of the larger circle is 14, calculate the radius of the smaller circle.

$c =$ $\boxed{5.5}$ **or** $\boxed{5\frac{1}{2}}$ **or** $\boxed{\dfrac{11}{2}}$

5. Determine exactly the <u>minimum</u> possible value of $c$ such that if $f(x)$ is a straight line passing through (1, 7), then $f(f(x))$ passes through (7, $c$).

$\boxed{2\sqrt{6}}$

6. A circle with center $C$ is internally tangent to $x^2 + y^2 = 16$ and externally tangent to $x^2 - 2x + y^2 = 0$.
The collection of all possible points $C$ form an ellipse.
Determine exactly the length of the ellipse's minor axis.

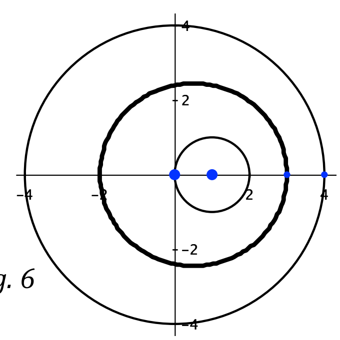


*Fig. 6*

# Minnesota State High School Mathematics League

## 2014-15 Meet 4, Team Event

## SOLUTIONS (page 2)

1. $f(3)=2\cdot f(2)+3\cdot f(1)=2\cdot f(2)+15$. *Also,* $f(4)=2\cdot f(3)+3\cdot f(2)$ *and* $265=2\cdot f(4)+3\cdot f(3)$. *Substituting the first equation into the second yields* $f(4)=2(2\cdot f(2)+15)+3\cdot f(2)=7\cdot f(2)+30$, *and substituting that result into the third equation yields* $265=2(7\cdot f(2)+30)+3(f(2)+15)=20\cdot f(2)+105$. *Solving for* $f(2)$, *we obtain* $f(2)=8$, *and then* $f(4)=7\cdot f(2)+30=7(8)+30=\boxed{86}$.

2. *Using the power of a point,* $PY\cdot PZ=PW\cdot PX \Rightarrow 3\cdot(3+YZ)=2\cdot 6 \Rightarrow YZ=1$. *Draw* $\overline{WZ}$, *creating right triangle PWZ.* $m\angle PZW=\tan^{-1}\frac{PW}{PZ}=\frac{1}{2}m\overset{\frown}{WY} \Rightarrow m\overset{\frown}{WY}=2\cdot\tan^{-1}\frac{1}{2}$. *Also,* $m\angle P=90°=\frac{1}{2}\left(m\overset{\frown}{XZ}-m\overset{\frown}{WY}\right) \Rightarrow m\overset{\frown}{XZ}=180°+m\overset{\frown}{WY}$, *so* $m\angle XOZ\ (\text{major})=180°+2\tan^{-1}\frac{1}{2}$, *and* $m\angle XOZ\ (\text{minor})=180°-2\tan^{-1}\frac{1}{2}$. *Now draw* $\overline{XZ}$, *creating right triangle XPZ, in which* $XZ=\sqrt{4^2+6^2}=2\sqrt{13}$, *and isosceles triangle XOZ. Drop an altitude from O to split* $\triangle XOZ$ *into two congruent right triangles. In each,* $\sin\left(\frac{1}{2}m\angle XOZ\right)=\frac{\sqrt{13}}{r} \Rightarrow r=\frac{\sqrt{13}}{\sin\left(90°-\tan^{-1}\frac{1}{2}\right)}=\frac{\sqrt{13}}{\sin\angle PWZ}=\frac{\sqrt{13}}{\frac{4}{2\sqrt{5}}}=\boxed{\frac{\sqrt{65}}{2}}$.

3. *We are given that the first figure has a perimeter of 10. Each new hexagon we attach will "cover" two sides of the previous figure's perimeter, so attaching one hexagon will reduce the previous perimeter by 2, but then also add on 4 additional new exterior sides for a net perimeter gain of +2. Attaching two hexagons will also reduce the previous perimeter by 2, but also add on 8 additional new exterior sides for a net gain of +6. Thus, every two figures we advance through the sequence, the perimeter will increase by +8. This reveals an arithmetic sub-sequence between odd-numbered figures, with* $a_1=10$ *and d = 8. 2015 is the 1008*$^{th}$ *odd number, so the 2015*$^{th}$ *figure of the original sequence is the 1008*$^{th}$ *term of the sub-sequence. Hence, the perimeter of the 2015*$^{th}$ *figure is* $a_1+d(n-1)=10+8(1008-1)=\boxed{8066}$.

4. *Let S be the center of the smaller circle, and L the center of the larger circle. Draw radii from each the centers to point A, then drop an altitude from A to meet the segment connecting the centers, at G (see Figure 4). This creates two right triangles:* $\triangle AGL$, *where* $\sin 20°=\frac{AG}{14} \Rightarrow AG=14\cdot\sin 20°$, *and* $\triangle AGS$, *where* $\sin 85°=\frac{AG}{r} \Rightarrow r=\frac{AG}{\sin 85°}=\boxed{\frac{14\cdot\sin 20°}{\sin 85°}}$.

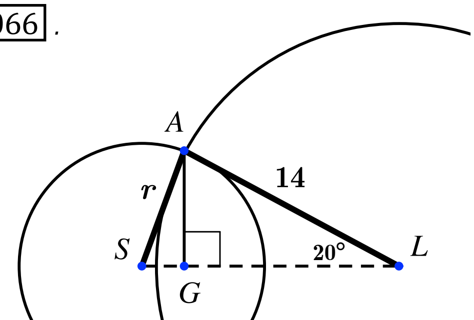


*Figure 4*

5. *Because* $f(x)$ *is linear, let* $f(x)=ax+b$. *Since* $f(x)$ *passes through the point* $(1,7)$, *we have* $7=a+b \Rightarrow b=7-a$, *and* $f(f(x))=a(ax+b)+b=a^2x+ab+b$. *Substituting for b yields the expression* $a^2x+a(7-a)+(7-a)$, *and setting x = 7,* $f(f(7))=a^2(7)+a(7-a)+(7-a)=6a^2+6a+7=c$. *To find the* <u>*minimum*</u> *possible value of c, complete the square on the quadratic expression:* $6a^2+6a+7=6\left(a+\frac{1}{2}\right)^2-\frac{6}{4}+7=6\left(a+\frac{1}{2}\right)^2+\frac{11}{2}$. *Therefore, c =* $\boxed{5.5}$.

6. *The first circle is centered at (0, 0) with radius 4, while the second is centered at (1, 0) with radius 1. The foci of the ellipse are the circles' centers, so the ellipse is centered at (0.5, 0). Using (3, 0) as one possible point C, the constant sum of distances in this ellipse is 3 + 2 = 5. Thus c = 0.5, a = 2.5, and* $c^2=a^2-b^2 \Rightarrow b=\sqrt{2.5^2-0.5^2}=\sqrt{6}$. *Minor axis =* $\boxed{2\sqrt{6}}$.