Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

______________ 1. Determine exactly the value of $\frac{LCM(20,14)}{GCD(20,14)}$.

______________ 2. The expression $5-\cfrac{1}{4-\cfrac{1}{3-\cfrac{1}{2-\cfrac{1}{1}}}}$ can be simplified and written as a single rational number.

Determine exactly that rational number.

Day # ______________ 3. The ePad was originally priced at $100, but the newest model of ePad is coming out, so the old one is going on sale. Each day, the price is reduced by 10%, and rounded <u>down</u> to the nearest dollar, as necessary. (On Day 1, the price is reduced to $90; on Day 2 it is $81; on Day 3 it is $72, and so on.) What is the <u>first</u> day on which the old ePad will cost $1 ?

______________ 4. Express $\frac{1}{9}+\frac{1}{11}+\frac{1}{999}$ as a repeating decimal.

Name: ______________________________ Team: ______________________________

Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event A

SOLUTIONS

NO CALCULATORS are allowed on this event.

$\boxed{70}$

1. Determine exactly the value of $\dfrac{LCM(20,14)}{GCD(20,14)}$.

$$\frac{LCM(20,14)}{GCD(20,14)} = \frac{LCM(2\cdot 2\cdot 5,\ 2\cdot 7)}{GCD(2\cdot 2\cdot 5,\ 2\cdot 7)} = \frac{\cancel{2}\cdot 2\cdot 5\cdot 7}{\cancel{2}} = 70.$$

$\boxed{\dfrac{33}{7}}$

2. The expression $5-\cfrac{1}{4-\cfrac{1}{3-\cfrac{1}{2-\cfrac{1}{1}}}}$ can be simplified and written as a single rational number.

Determine exactly that rational number.

Work from the inside out: $5-\cfrac{1}{4-\cfrac{1}{3-\cfrac{1}{2-\cfrac{1}{1}}}} = 5-\cfrac{1}{4-\cfrac{1}{3-\cfrac{1}{1}}} = 5-\cfrac{1}{4-\cfrac{1}{2}} = 5-\cfrac{1}{\left(\cfrac{7}{2}\right)} = 5-\frac{2}{7} = 4\frac{5}{7}.$

Day # $\boxed{27}$

3. The ePad was originally priced at \$100, but the newest model of ePad is coming out, so the old one is going on sale. Each day, the price is reduced by 10%, and rounded <u>down</u> to the nearest dollar, as necessary. (On Day 1, the price is reduced to \$90; on Day 2 it is \$81; on Day 3 it is \$72, and so on.) What is the <u>first</u> day on which the old ePad will cost \$1 ?

The basic idea is that we're subtracting the tens place of the previous day's cost, along with an additional dollar if the ones place is greater than zero. (For example, 72 – 7.2 rounds to 72 – 8 = 64.) Make a table:

Day #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**
Cost(\$)	**90**	**81**	**72**	**64**	**57**	**51**	**45**	**40**	**36**	**32**	**28**	**25**	**22**	**19**	**17**	**15**	**13**	**11**	**9**	**8**	**7**	**6**	**5**	**4**	**3**	**2**	**1**

$\boxed{0.\overline{203021}}$

4. Express $\dfrac{1}{9}+\dfrac{1}{11}+\dfrac{1}{999}$ as a repeating decimal.

$\dfrac{1}{9}=0.\overline{1},\ \dfrac{1}{11}=0.\overline{09},\ \dfrac{1}{999}=0.\overline{001}.$ *We have repetends of lengths 1, 2, and 3, so to add them, we need to extend to a common multiple: 6 decimal places:*

$$\left(0.\overline{111111}+0.\overline{090909}\right)+0.\overline{001001}=0.\overline{202020}+0.\overline{001001}=0.\overline{203021}$$

Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

______________ 1. If $\frac{1}{3}$ and $\frac{1}{4}$ are the lengths of the two legs of a right triangle, determine exactly the length of the triangle's hypotenuse.

$m\angle APQ =$ ______________ 2. In *Figure 2*, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{PQ}$ is a transversal, $m\angle BPQ = 2x$, and $m\angle DQP = 5x - 51$. Determine exactly the measure of angle *APQ*.

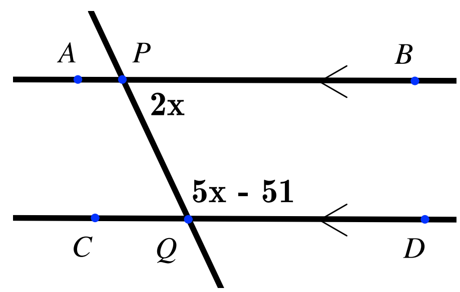


Figure 2

$m\angle FKH =$ ______________ 3. In isosceles triangle *FGH (Figure 3)*, the vertex angle measures 20° and its opposite side is 20 cm long. A segment from point *H* meets side $\overline{FG}$ at point *K* so that *HK* = 20 cm. Determine the measure of angle *FKH*.

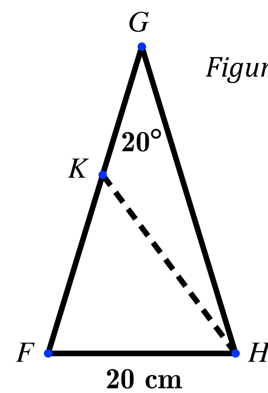


Figure 3

$m\angle WVZ =$ ______________ 4. In *Figure 4*, *XY* = *XZ*, $m\angle YXV = 30°$, and *XV* = *XW*. Determine exactly the measure of angle *WVZ.*

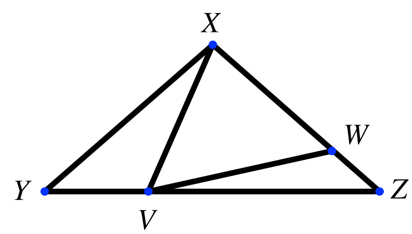


Figure 4

Name: ______________________________ Team: ______________________________

Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event B

SOLUTIONS

$\boxed{\frac{5}{12}}$ *or* $\boxed{0.41\overline{6}}$

1. If $\frac{1}{3}$ and $\frac{1}{4}$ are the lengths of the two legs of a right triangle, determine exactly the length of the triangle's hypotenuse.

$$a^2+b^2=c^2 \Rightarrow \left(\frac{1}{3}\right)^2+\left(\frac{1}{4}\right)^2=c^2 \Rightarrow \frac{1}{9}+\frac{1}{16}=c^2 \Rightarrow c=\sqrt{\frac{25}{144}}=\frac{5}{12}.$$

$m\angle APQ = \boxed{114^\circ}$

2. In *Figure 2*, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{PQ}$ is a transversal, $m\angle BPQ = 2x$, and $m\angle DQP = 5x-51$. Determine exactly the measure of angle *APQ*.

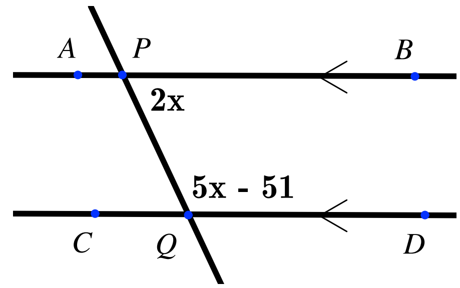


Figure 2

$\angle BPQ$ and $\angle DQP$ are same-side interior angles, so they are supplementary: $2x+(5x-51)=180$

$7x=231 \Rightarrow x=33$*, and since $\angle APQ$ and $\angle DQP$ are congruent (alternate interior angles),*

$m\angle APQ = 5x-51 = 5(33)-51 = 165-51 = 114^\circ$.

$m\angle FKH = \boxed{80^\circ}$

3. In isosceles triangle *FGH (Figure 3)*, the vertex angle measures 20° and its opposite side is 20 cm long. A segment from point *H* meets side $\overline{FG}$ at point *K* so that *HK* = 20 cm. Determine the measure of angle *FKH*.

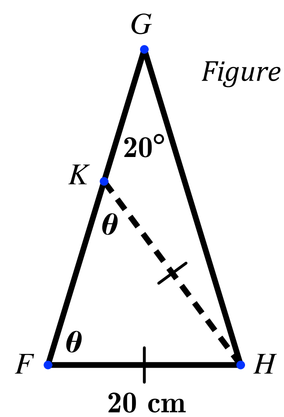


Figure 3

ΔHFK is isosceles, so label $m\angle FKH = m\angle F = \theta$. We are told that ΔFGH is isosceles, so $m\angle FHG = \theta$ also. Considering the three angles in ΔFGH, $20+\theta+\theta=180 \Rightarrow \theta=80^\circ$.

$m\angle WVZ = \boxed{15^\circ}$

4. In *Figure 4*, *XY* = *XZ*, $m\angle YXV = 30^\circ$, and *XV* = *XW*. Determine exactly the measure of angle *WVZ*.

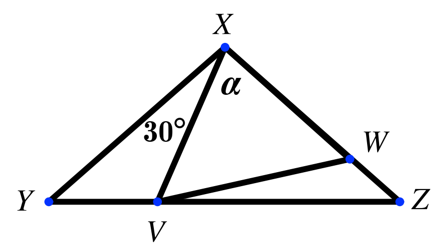


Figure 4

Let $m\angle VXW = \alpha$. Then $m\angle YXZ = 30^\circ+\alpha$, and

$m\angle Z = \frac{1}{2}\left[180^\circ - (30^\circ+\alpha)\right] = 75^\circ - \frac{\alpha}{2}$.

Also, $m\angle VWX = \frac{1}{2}(180^\circ-\alpha) = 90^\circ - \frac{\alpha}{2}$. *$\angle VWX$ is exterior to ΔVWZ, so* $m\angle VWX = m\angle WVZ + m\angle Z$

$\Rightarrow 90^\circ - \frac{\alpha}{2} = m\angle WVZ + \left(75^\circ - \frac{\alpha}{2}\right) \Rightarrow m\angle WVZ = 15^\circ$.

Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

$a =$ ______________

1. In *Figure 1*, point *P(a, b)* is located in the second quadrant on the circumference of a circle of radius r. Express a in terms of b and r.

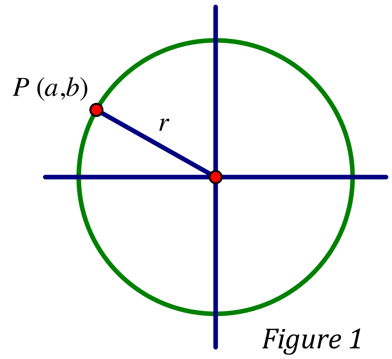


Figure 1

$\sin\theta =$ ______________

2. A line segment drawn from the origin to a point (m, n) in the fourth quadrant makes an angle of $\boldsymbol{\theta}$ with the positive x-axis. Express $\sin\boldsymbol{\theta}$ in terms of m and n.

$\tan\alpha =$ ______________

3. The point $(\boldsymbol{\alpha}, k)$ lies on the graph of $y = \cos x$ (*Figure 3*), beneath the y-axis and with $\boldsymbol{\alpha}$ between 4 and 5 radians. Express $\tan\boldsymbol{\alpha}$ in terms of k.

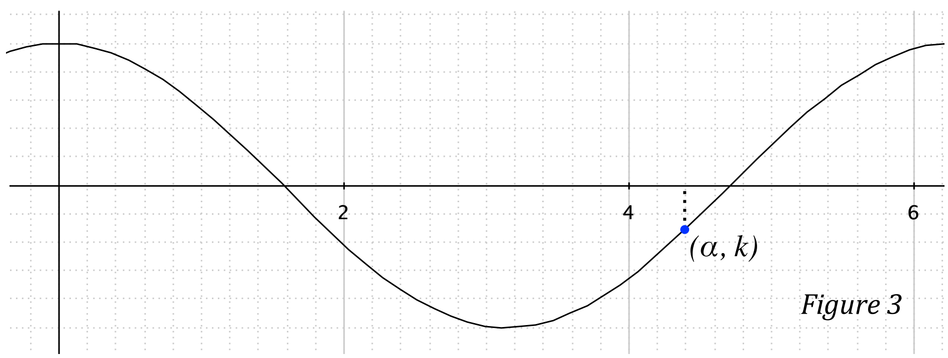


Figure 3

$BD =$ ______________

4. Semicircle O, pictured in *Figure 4*, has diameter $AB = 10$. Arc $\overset{\frown}{AC}$ also has length 10. From C, a perpendicular is dropped to meet <u>segment</u> $\overline{AB}$ at D. Calculate the length BD.

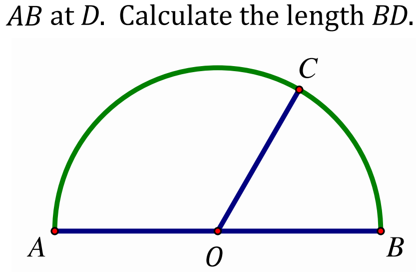


Figure 4

Name: ______________________________ Team: ______________________________

Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event C

SOLUTIONS

$a = \boxed{-\sqrt{r^2-b^2}}$

1. In *Figure 1*, point *P(a, b)* is located in the second quadrant on the circumference of a circle of radius *r*. Express *a* in terms of *b* and *r*.

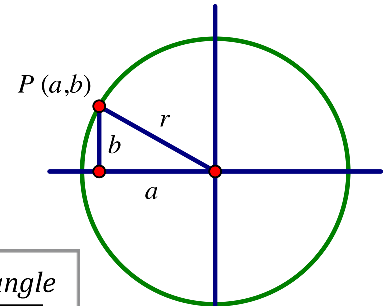


Figure 1

Drop a perpendicular from P to the x-axis as shown, creating a right triangle with legs a, b and hypotenuse r. $a^2+b^2=r^2 \Rightarrow a^2=r^2-b^2 \Rightarrow a=\pm\sqrt{r^2-b^2}$, *and since a lies on the negative x-axis,* $a=-\sqrt{r^2-b^2}$.

$\sin\theta = \boxed{\dfrac{n}{\sqrt{m^2+n^2}}}$

or $\boxed{\dfrac{n\sqrt{m^2+n^2}}{m^2+n^2}}$

or equivalent

2. A line segment drawn from the origin to a point (m, n) in the fourth quadrant makes an angle of $\boldsymbol{\theta}$ with the positive x-axis. Express sin $\boldsymbol{\theta}$ in terms of m and n.

See Figure 2. $\sin\theta=\dfrac{opp}{hyp}=\dfrac{n}{c}$, *and since* $m^2+n^2=c^2$, *we have* $c=\sqrt{m^2+n^2} \Rightarrow \sin\theta=\dfrac{n}{\sqrt{m^2+n^2}}=\dfrac{n\sqrt{m^2+n^2}}{m^2+n^2}$.

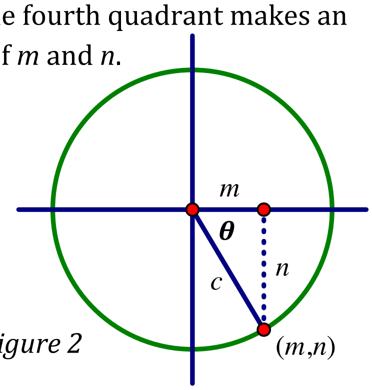


Figure 2

$\tan\alpha = \boxed{\dfrac{-\sqrt{1-k^2}}{k}}$

3. The point ($\boldsymbol{\alpha}$, k) lies on the graph of $y = \cos x$ (*Figure 3*), beneath the y-axis and with $\boldsymbol{\alpha}$ between 4 and 5 radians. Express tan $\boldsymbol{\alpha}$ in terms of k.

$\cos\alpha=k$, *which is negative.*

$\pi<\alpha<\dfrac{3\pi}{2}$, *so* $\sin\alpha$ *is also negative.*

$\sin^2\alpha+\cos^2\alpha=1 \Rightarrow \sin\alpha=\pm\sqrt{1-\cos^2\alpha}$

$=\pm\sqrt{1-k^2}$. $\tan\alpha=\dfrac{\sin\alpha}{\cos\alpha}=\dfrac{-\sqrt{1-k^2}}{k}$.

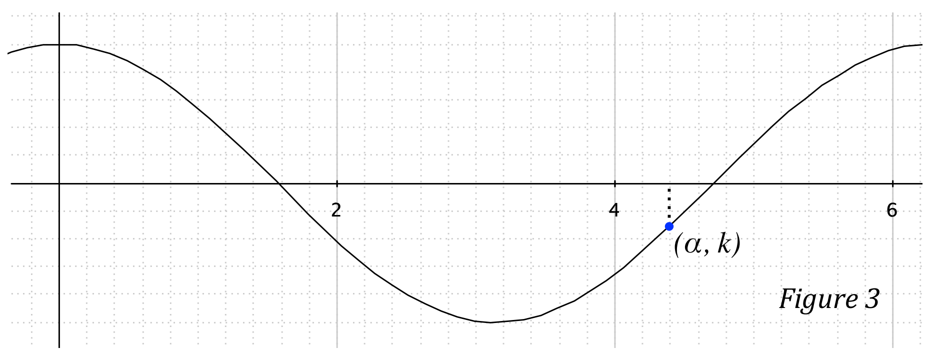


Figure 3

$BD = \boxed{5+5\cos(2)}$

or $\boxed{\approx 2.919}$

Graders: if students use $\dfrac{360°}{\pi}$ as an angle equivalent, the degree sign <u>must</u> be included.

4. Semicircle *O*, pictured in *Figure 4*, has diameter *AB* = 10. Arc $\widehat{AC}$ also has length 10. From *C*, a perpendicular is dropped to meet <u>segment</u> $\overline{AB}$ at *D*. Calculate the length *BD*.

Since $\widehat{AC}$ *has a length equal to twice the semicircle's radius, by definition of a "radian", angle AOC measures exactly 2 radians.* $\cos\angle COD=\dfrac{OD}{5}$ *and* $\cos\angle COA=\cos(2)$, *but also, since* $\angle COD$ *and* $\angle COA$ *are supplementary, their cosines are opposites.* $OD=-5\cos(2) \Rightarrow BD=OB-OD=5-(-5\cos(2))$.

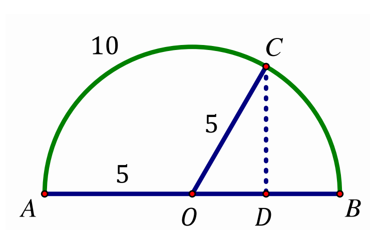


Figure 4

Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

$r_2 =$ ________________

1. Let r_1 and r_2 be the distinct roots of $r^2 - r - 20$, with $r_1 < r_2$. Determine r_2 exactly.

$\frac{1}{x_1} + \frac{1}{x_2} =$ ________________

2. Let x_1 and x_2 be the solutions of $x^2 - 20x + 13 = 0$. Determine $\frac{1}{x_1} + \frac{1}{x_2}$ exactly.

$f(-1) =$ ________________

3. Let $f(x)$ be a quadratic polynomial. If it is known that $f(0) = 1$, $f(1) = 2$, and $f(2) = 0$, determine $f(-1)$ exactly.

$A + B + C =$ ________________

4. Let $A(X - B)(X - C) = (X - 1)(X + 3) + (X - 2)(3X - 2) + (X + 3)(2X - 1)$, where A, B, and C are all real numbers. Determine the sum $A + B + C$ exactly.

Name: ________________________ Team: ________________________

Minnesota State High School Mathematics League

2013-14 Meet 1, Individual Event D

SOLUTIONS

NO CALCULATORS are allowed on this event.

$r_2 = \boxed{5}$

1. Let r_1 and r_2 be the distinct roots of $r^2 - r - 20$, with $r_1 < r_2$. Determine r_2 exactly.

$r^2 - r - 20 = (r-5)(r+4) = 0 \Rightarrow r = -4$ or 5. Since $-4 < 5$, $r_1 = -4$ and $r_2 = 5$.

$\frac{1}{x_1} + \frac{1}{x_2} = \boxed{\frac{20}{13}}$ **or** $\boxed{1\frac{7}{13}}$

2. Let x_1 and x_2 be the solutions of $x^2 - 20x + 13 = 0$. Determine $\frac{1}{x_1} + \frac{1}{x_2}$ exactly.

Of course we could use the quadratic formula to find both roots, but it's more elegant to use the formulas for the sum & product of roots: $\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_2}{x_1 x_2} + \frac{x_1}{x_1 x_2} = \frac{x_1 + x_2}{x_1 x_2} = \frac{\text{sum of roots}}{\text{product of roots}} = \frac{20}{13}$.

$f(-1) = \boxed{-3}$

3. Let $f(x)$ be a quadratic polynomial. If it is known that $f(0) = 1$, $f(1) = 2$, and $f(2) = 0$, determine $f(-1)$ exactly.

Let $f(x) = ax^2 + bx + c$. *Then* $f(0) = a(0)^2 + b(0) + c \Rightarrow c = 1$, $f(1) = a(1)^2 + b(1) + c = 2$

$\Rightarrow a + b + 1 = 2 \Rightarrow a + b = 1$, *and* $f(2) = a(2)^2 + b(2) + c = 4a + 2b + c = 2a + 2(a+b) + c = 0$

$\Rightarrow 2a + 2 + 1 = 0 \Rightarrow a = \frac{-3}{2}, b = \frac{5}{2}$. *Thus our quadratic polynomial is* $f(x) = -\frac{3}{2}x^2 + \frac{5}{2}x + 1$,

and we have $f(-1) = -\frac{3}{2}(-1)^2 + \frac{5}{2}(-1) + 1 = -\frac{3}{2} - \frac{5}{2} + 1 = -\frac{8}{2} + 1 = -4 + 1 = -3$.

$A + B + C = \boxed{\frac{37}{6}}$ **or** $\boxed{6\frac{1}{6}}$

4. Let $A(X-B)(X-C) = (X-1)(X+3) + (X-2)(3X-2) + (X+3)(2X-1)$, where A, B, and C are all real numbers. Determine the sum $A + B + C$ exactly.

Rearrange the right side of the equation, then factor out $(X+3)$ *from two of the terms:*

$(X+3)\left[(X-1) + (2X-1)\right] + (X-2)(3X-2) = (X+3)(3X-2) + (X-2)(3X-2) = (2X+1)(3X-2)$.

Since the coefficient of the X term in each factor must be 1, we factor 2 out of the first factor and 3 out of the second factor to yield: $2\left(X + \frac{1}{2}\right) \cdot 3\left(X - \frac{2}{3}\right) = 6\left(X + \frac{1}{2}\right)\left(X - \frac{2}{3}\right) = A(X-B)(X-C)$.

So $A = 6, B = -\frac{1}{2}, C = \frac{2}{3} \Rightarrow A + B + C = 6 - \frac{1}{2} + \frac{2}{3} = 6 - \frac{3}{6} + \frac{4}{6} = 6\frac{1}{6}$.

Minnesota State High School Mathematics League

2013-14 Meet 1, Team Event

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

sum = ________

1. Find the sum of <u>all</u> positive integers n for which LCM(2, n) = GCD(n, 210).

$m\angle B =$ ________

2. On isosceles $\triangle ABC$, points P and Q are on sides $\overline{BC}$ and $\overline{AB}$ respectively, distinct from A, B, and C, so that $AC = AP = PQ = QB$. Determine exactly the measure of angle B.

AB = ________

3. *Figure 3* shows a semicircle with radius $CA = 1$. If arc $\overset{\frown}{AB}$ also has length 1, calculate the length of chord $\overline{AB}$.

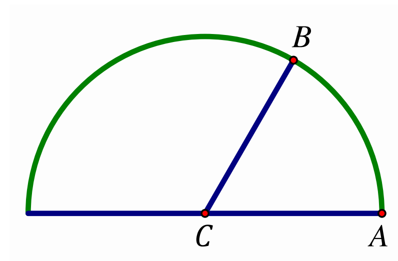


Figure 3

a = ________

4. Let $f(t) = t^2 + at - 1$. Given that at least one of the coefficients of the degree-four polynomial $f(f(t))$ is zero, list all possible values of a.

k = ________

5. *Figure 5* shows rays drawn from the origin at angular intervals of 10°, intersecting the line $x = 1$ at y_1, y_2, etc. Find the <u>smallest</u> positive integer k for which $y_k - y_{k-1}$ will equal or exceed $2y_1$.

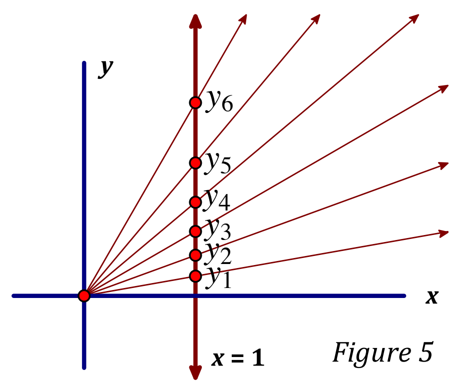


Figure 5

b = ________

6. Let N be a number in base b such that $N_b = 14_b \cdot 17_b$. What is the <u>greatest</u> base b for which N_b would be written with "2" as its left-most digit?

Team: ______________________

Minnesota State High School Mathematics League

2013-14 Meet 1, Team Event

SOLUTIONS (page 1)

sum = **384**

1. Find the sum of <u>all</u> positive integers n for which LCM(2, n) = GCD(n, 210).

$m\angle B = \boxed{\left(25\frac{5}{7}\right)^\circ}$ **or** $\boxed{\left(\frac{180}{7}\right)^\circ}$

2. On isosceles $\triangle ABC$, points P and Q are on sides $\overline{BC}$ and $\overline{AB}$ respectively, distinct from A, B, and C, so that $AC = AP = PQ = QB$. Determine exactly the measure of angle B.

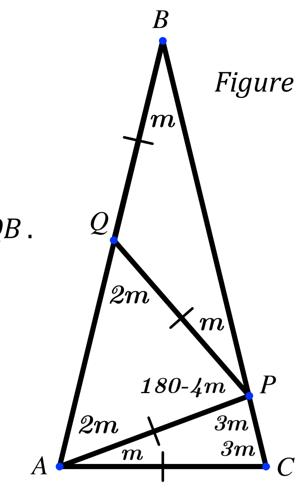
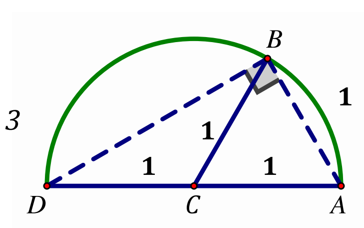


Figure 2

$AB \approx \boxed{0.959}$ **or** $\boxed{0.958}$ **or** $\boxed{2\cdot\sin(0.5)}$ **or** $\boxed{\sqrt{2-2\cos(1)}}$

3. *Figure 3* shows a semicircle with radius $CA = 1$. If arc $\overset{\frown}{AB}$ also has length 1, calculate the length of chord $\overline{AB}$.

Figure 3



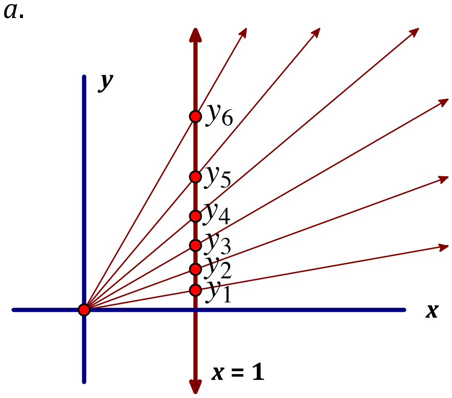
$a = \boxed{-2, 0, 1, 2}$

Graders: award 1 point per correct value

4. Let $f(t) = t^2 + at - 1$. Given that at least one of the coefficients of the degree-four polynomial $f(f(t))$ is zero, list all possible values of a.

$k = \boxed{5}$

5. *Figure 5* shows rays drawn from the origin at angular intervals of 10°, intersecting the line $x = 1$ at y_1, y_2, etc. Find the <u>smallest</u> positive integer k for which $y_k - y_{k-1}$ will equal or exceed $2y_1$.



$b = \boxed{13}$

6. Let N be a number in base b such that $N_b = 14_b \cdot 17_b$. What is the <u>greatest</u> base b for which N_b would be written with "2" as its left-most digit?

Minnesota State High School Mathematics League

2013-14 Meet 1, Team Event

SOLUTIONS (page 2)

1. *For LCM(2, n) = GCD(n, 210), both quantities must be equal to n. This means n is both a multiple of 2 and a factor of 210. Examining the prime factorization of 210 (* $2\cdot3\cdot5\cdot7$ *), the acceptable values of n must be of the form* $2\cdot3^a\cdot5^b\cdot7^c$*, where a, b, and c are all either 0 or 1. This yields eight values: 2, 6, 10, 14, 30, 42, 70, and 210. Their sum is* $\boxed{384}$.

2. *See Figure 2. Begin by labeling* $m\angle B$ *as m. Then in isosceles* $\triangle PQB$, $m\angle P = m$ *also.* $\angle PQA$ *is an exterior angle of this triangle, and so its measure is 2m. Using isosceles* $\triangle QPA$, $m\angle A = 2m$*, and* $m\angle QPA = 180-4m$*. Subtracting along straight angle CPB,* $m\angle CPA = 3m$*, and in isosceles* $\triangle CPA$, $m\angle C = 3m$ *also. Now we must remember that the original triangle,* $\triangle ABC$*, was given as isosceles, so* $m\angle QAC = 3m = 2m + m\angle PAC \Rightarrow m\angle PAC = m$*. We now have expressions for all angle measures in* $\triangle CPA$: $m+3m+3m = 180° \Rightarrow m = \boxed{\left(\frac{180}{7}\right)^\circ}$.

3. *Drawing segments* $\overline{BD}$ *and* $\overline{BA}$ *creates* $\triangle DBA$*, and because it is inscribed in a semicircle, it is a right triangle with right angle at B. Because arc* $\overset{\frown}{AB}$ *has the same length as its circle's radius, by the definition of "radian",* $m\angle ACB$ *= 1 radian, and* $m\angle D$ *= 0.5 radians. Using trigonometry,* $\sin(0.5) = \frac{AB}{2} \Rightarrow AB = 2\sin(0.5) \approx \boxed{0.959}$.

4. $f(t^2+at-1) = (t^2+at-1)^2 + a(t^2+at-1) - 1 = \left[t^4+2at^3+(a^2-2)t^2-2at+1\right] + (at^2+a^2t-a) - 1$
$= t^4+2at^3+(a^2+a-2)t^2+(a^2-2a)t-a$*. Since one of the coefficients is zero, our possibilities are* $2a=0 \Rightarrow a=0$; $a^2+a-2=0 \Rightarrow (a+2)(a-1)=0 \Rightarrow a=-2$ *or* 1; $a^2-2a=0 \Rightarrow a(a-2)=0 \Rightarrow a=0$ *or* 2 *(only 2 is unique), or a = 0 (repeat value again). The unique values of a are* $\boxed{-2, 0, 1, 2}$.

5. $y_k - y_{k-1} = \left(\frac{y_1+y_2+\ldots+y_k}{1}\right) - \left(\frac{y_1+y_2+\ldots+y_{k-1}}{1}\right) = \tan(10k)^\circ - \tan(10(k-1))^\circ$. $y_1 = \tan 10° \approx 0.1763$*, so* $2y_1 \approx 0.3526$*. Therefore, we want* $\tan(10k)^\circ - \tan(10(k-1))^\circ \geq 0.3526$*. Generate a table of possible values for k:*

k	1	2	3	4	5	6
$\tan(10k)^\circ - \tan(10(k-1))^\circ$	≈ 0.1763	≈ 0.1876	≈ 0.2134	≈ 0.2617	≈ 0.3527	≈ 0.5403

The first value that satisfies the inequality is k = $\boxed{5}$.
(It turns out, in fact, that $\tan 50° - \tan 40° = 2\cdot\tan 10°$*. Can you* <u>prove</u> *why this is true?)*

6. $N_b = 14_b \cdot 17_b \Rightarrow N = (b+4)(b+7) = b^2+11b+28$*. Because we want a quantity of 2 in the "* b^2 *" column, we need* $11b+28 \geq b^2$ *(and also* $11b+28 < 2b^2$ *). Either solve the inequality or try values of b > 10 to find the greatest b =* $\boxed{13}$.