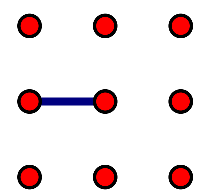
Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have an extended 20 minutes for this event.

________________ 1. Jill's father looked at the money on the table. "You've got quite a lot there," he remarked.
"I found two dollars on the sidewalk," the girl explained.
Her father laughed. "You were lucky. Now you've got five times as much as you'd have had if you'd *lost* two bucks."
How much did Jill have before her lucky find?

________________ 2. Two players play a game using the 3 x 3 grid of dots shown in *Figure 2*. The players alternate turns, where each turn involves drawing a single horizontal or vertical line segment that connects two adjacent dots. (A sample turn is shown.) The first player to completely enclose any 1-unit square loses. What is the greatest number of turns a game can last?



(Figure 2)

________________ 3. "So, you just turned forty," I said to my neighbor. "How old are your three kids now?"
"Figure it out yourself," he said. "Their three ages add up to the number of my street address, and multiply to make my age."
After a moment, I replied, "Okay, I got it... but only because I know you don't have twins."
What were the ages of the three children?

________________ 4. If my three were a four, and my one were a three,
What I am would be nine less than half of what I'd be.
I'm only three digits, just three in a row...
So what number am I? Think it over: do you know?

Name: ______________________________ Team: ______________________________

Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event A

SOLUTIONS

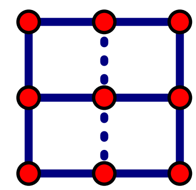
$3.00

1. Jill's father looked at the money on the table. "You've got quite a lot there," he remarked.
"I found two dollars on the sidewalk," the girl explained.
Her father laughed. "You were lucky. Now you've got five times as much as you'd have had if you'd *lost* two bucks."
How much did Jill have before her lucky find? ***(Hunter's Math Brain Teasers)***

Suppose Jill had \$X before her find. Then $X+2=5(X-2) \Rightarrow 12=4X \Rightarrow X=3$.

11

2. Two players play a game using the 3 x 3 grid of dots shown in *Figure 2*. The players alternate turns, where each turn involves drawing a single horizontal or vertical line segment that connects two adjacent dots. (A sample turn is shown.) The first player to completely enclose any 1-unit square loses. What is the greatest number of turns a game can last?



(Figure 2)

First draw all 12 legal line segments. This creates four 1-unit squares. Removing any exterior line segment "undoes" only one square, while removing an interior line segment undoes two adjacent squares. To undo all 4 squares, remove two interior segments, leaving 10 segments. The 11th segment drawn will, of necessity, enclose a 1-unit square.

8, 5, and 1

(Graders: must have all three ages, in any order, to receive credit)

3. "So, you just turned forty," I said to my neighbor. "How old are your three kids now?"
"Figure it out yourself," he said. "Their three ages add up to the number of my street address, and multiply to make my age."
After a moment, I replied, "Okay, I got it... but only because I know you don't have twins."
What were the ages of the three children? ***(Hunter's Math Brain Teasers)***

First note that $40=1\cdot 2\cdot 2\cdot 2\cdot 5$. *The possibilities for the ages are* {20, 2, 1}, {10, 4, 1}, {10, 2, 2}, {8, 5, 1}, and {5, 4, 2}. *These have sums of* 23, 15, 14, 14, *and* 11 *respectively. Since "no twins" was a key fact, the house number must have been* 14, *eliminating* {10, 2, 2} *in favor of* {8, 5, 1}.

183

4b. If my three were a four, and my one were a three,
What I am would be nine less than half of what I'd be.
I'm only three digits, just three in a row...
So what number am I? Think it over: do you know? ***(Hunter's Math Brain Teasers)***

Investigate the last digit. If it is the "three", then it is 9 less than a number ending in 2, which would be half of a number ending in four. This fits the given description. Now, where to put the "one"? Certainly the first digit; if the first digit didn't change, the "half" relationship would be problematic. We now have $\underline{1}\,\underline{T}\,\underline{3}=\frac{1}{2}\left(\underline{3}\,\underline{T}\,\underline{4}\right)-9 \Rightarrow 100+10T+3=\frac{1}{2}\left(300+10T+4\right)-9$

$\Rightarrow 100+10T+3=150+5T-7 \Rightarrow 5T=40 \Rightarrow T=8$, *and the original number is* 183.

Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

______________ 1. *Figure 1* shows equilateral triangle ABC, with $\overline{DE} \parallel \overline{BC}$. If $AD = 2$ and $BD = 3$, what is the ratio of the perimeter of $\triangle ADE$ to the perimeter of $\triangle ABC$?

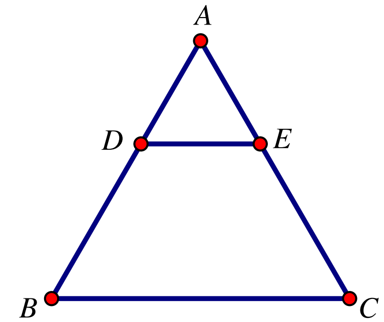


Figure 1

______________ 2. The area of circle O_1 is 19% less than the area of circle O_2. If O_1 has a diameter of length 36, what is the length of O_2's diameter?

______________ 3. *Figure 1* shows equilateral triangle ABC, with $\overline{DE} \parallel \overline{BC}$ and $BC = 8$. If $\triangle ADE$ and trapezoid $BDEC$ have equal perimeters, determine exactly the area of $\triangle ADE$.

______________ 4. In square $PQRS$ *(Figure 4)*, segments $\overline{QT}$ and $\overline{RU}$ are perpendicular and meet at V. If $RV = 3$ and $QV = 4$, determine exactly the area of quadrilateral $PQVU$.

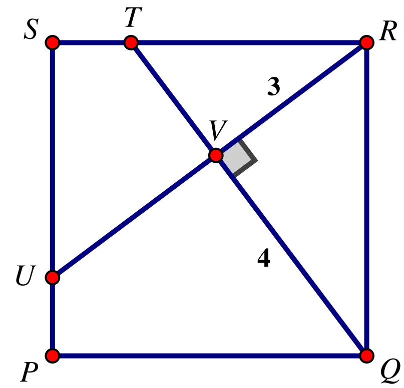


Figure 4

Name: ______________________ Team: ______________________

Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event B

SOLUTIONS

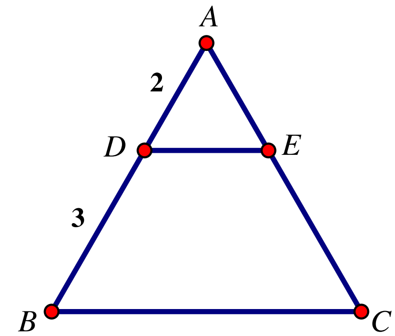


Figure 1

2:5

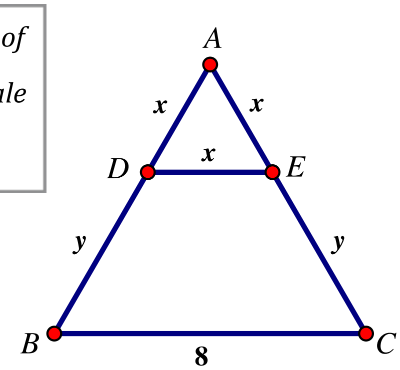
1. *Figure 1* shows equilateral triangle ABC, with $\overline{DE} \parallel \overline{BC}$. If $AD = 2$ and $BD = 3$, what is the ratio of the perimeter of $\triangle ADE$ to the perimeter of $\triangle ABC$?

$\triangle ABC \sim \triangle ADE$ by AA, so the ratio of the triangles' perimeters is the same as the ratio of corresponding side lengths. AD : AB = 2 : 5.

40

2. The area of circle O_1 is 19% less than the area of circle O_2. If O_1 has a diameter of length 36, what is the length of O_2's diameter?

Area[O_1] : Area[O_2] = 81 : 100, so the ratio of corresponding lengths of the two circles is $\sqrt{81} : \sqrt{100} = 9:10$. Multiplying this ratio by a scale factor of 4 reveals O_2's diameter as 40.



$9\sqrt{3}$

3. *Figure 1* shows equilateral triangle ABC, with $\overline{DE} \parallel \overline{BC}$ and $BC = 8$. If $\triangle ADE$ and trapezoid $BDEC$ have equal perimeters, determine exactly the area of $\triangle ADE$.

Let AD = x, BD = y. Then $x + x + x = y + x + y + 8 \Rightarrow 3x = x + 2y + 8 \Rightarrow y = x - 4$.

Since AB = AD + DB = 8, $x + (x-4) = 8 \Rightarrow x = 6$. *Area[$\triangle ADE$]* $= \dfrac{x^2\sqrt{3}}{4} = \dfrac{36\sqrt{3}}{4} = 9\sqrt{3}$.

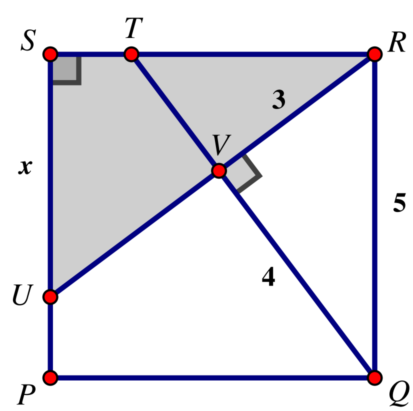


Figure 4

$\dfrac{77}{8}$

4. In square *PQRS (Figure 4)*, segments $\overline{QT}$ and $\overline{RU}$ are perpendicular and meet at *V*. If $RV = 3$ and $QV = 4$, determine exactly the area of quadrilateral *PQVU*.

There are many similar triangles in the figure, and many legitimate ways to solve the problem. A method which does not involve drawing any additional segments proceeds as follows:

$$\triangle RVQ \sim \triangle USR \Rightarrow \frac{RV}{US} = \frac{VQ}{SR} \Rightarrow \frac{3}{x} = \frac{4}{5} \Rightarrow x = \frac{15}{4}.$$

Area[PQVU] = Area[PQRS] – Area[$\triangle USR$] – Area[$\triangle RVQ$]

$$= 25 - \frac{1}{2}\cdot x \cdot 5 - \frac{1}{2}\cdot 3 \cdot 4 = 25 - \frac{75}{8} - 6 = \frac{77}{8}.$$

Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

__________________ 1. Katie has 5 pairs of earrings mixed together in a jewelry bag. She does not have time to look through and pick out two matching earrings. What is the ***fewest*** number of earrings she has to grab to be certain she has a matching pair?

__________________ 2. An ordinary deck of 52 playing cards is shuffled and 2 cards are dealt face up. Calculate the probability that at least one of these is a spade.

__________________ 3. The rules of a particular board game state that if a playing piece is at the point (x, y) on the coordinate plane, then it is allowed to move to either $(x + 1,\ y)$ or $(x,\ y + 1)$.
If the playing piece starts at (0, 0) and moves randomly, ending up at (4, 4), calculate the probability that it has passed through (2, 2).

__________________ 4. In how many ways can 10 distinct math team members be assigned to ride to a meet in 3 different school vehicles (a maxi-van, a mini-bus, and a full-sized school bus), if each vehicle must have at least one math team member riding in it?

Name: __________________________ Team: __________________________

Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event C

SOLUTIONS

Answer: 6

1. Katie has 5 pairs of earrings mixed together in a jewelry bag. She does not have time to look through and pick out two matching earrings. What is the ***fewest*** number of earrings she has to grab to be certain she has a matching pair? ***[Mathematics Teacher, May 2004]***

Katie can choose 5 lone earrings, but then the 6th earring she selects must make a pair. (This is an excellent example of the ***Pigeonhole Principle****.)*

Answer: $\frac{15}{34}$ *or* ≈ 0.441 *or* $\approx 44.1\%$

2. An ordinary deck of 52 playing cards is shuffled and 2 cards are dealt face up. Calculate the probability that at least one of these is a spade. ***[NYCC, Fall 1984]***

Consider the probability that neither card is a spade: $\frac{39}{52}\cdot\frac{38}{51}=\frac{3}{4}\cdot\frac{38}{51}=\frac{19}{34}$.

This is the complement of P(at least 1 is a spade), so we take $1-\frac{19}{34}=\frac{15}{34}$.

Answer: $\frac{18}{35}$ *or* ≈ 0.514 *or* $\approx 51.4\%$

3. The rules of a particular board game state that if a playing piece is at the point (x, y) on the coordinate plane, then it is allowed to move to either $(x+1, y)$ or $(x, y+1)$. If the playing piece starts at (0, 0) and moves randomly, ending up at (4, 4), calculate the probability that it has passed through (2, 2). ***[NYCC, Spring 1977]***

To get from (0, 0) *to* (2, 2) *requires two "upward" moves and two "rightward" moves, in some order. There are four moves, and you must choose where in the sequence to place the two upward moves. There are* $\binom{4}{2}=6$ *ways to do this. By the same reasoning, there are* 6 *ways to move from* (2, 2) *to* (4, 4)*. This makes* 6 • 6 = 36 *ways to pass through* (2, 2)*. Overall, there are* $\binom{8}{4}=70$ *ways to get from* (0, 0) *to* (4, 4)*, so the probability is* $\frac{36}{70}=\frac{18}{35}$.

Answer: 55980

4. In how many ways can 10 distinct math team members be assigned to ride to a meet in 3 different school vehicles (a maxi-van, a mini-bus, and a full-sized school bus), if each vehicle must have at least one math team member riding in it?

For each student, there are 3 choices for which vehicle to ride in. This creates 3^{10} *possible riding arrangements, but we must now subtract those where at least one vehicle is empty. There are* 2^{10} *ways to distribute riders among only two of the vehicles, leaving one empty, and 3 choices for which vehicle to leave empty. Using the principle of inclusion/exclusion, we take* 3^{10}*, subtract* $3\cdot 2^{10}$*, then add back in the 3 arrangements where two vehicles are empty. The total is* 55980.

Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

________ 1. Evaluate $\left[a-\left(b-a\right)\right]\left[a+\left(b-a\right)\right]$ if $a = 3$ and $b = -2$.

________ 2. Tanea randomly selects a number from the set {1, 2, 3, 4, 5, 6, 7, 8, 9}. Antoine then randomly selects a number (not necessarily distinct) from the same set. Determine exactly the probability that Tanea's number is greater than Antoine's number.

$r =$ ________ 3. Equiangular hexagon $ABCDEF$ has sides $AB = CD = EF = r$ and $BC = DE = FA = 2$. If the area of $\triangle ACE$ is $13\sqrt{3}$, determine exactly the value of r.

________ 4. A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently at random from among {north, south, east, west}. Determine exactly the probability that the frog's final position is no more than 1 meter from its starting position.

Name: ________________ Team: ________________

Minnesota State High School Mathematics League

2010-11 Meet 5, Individual Event D

SOLUTIONS

NO CALCULATORS are allowed on this event.

$\boxed{-16}$

[AMC 12A, #1]

1. Evaluate $\left[a-(b-a)\right]\left[a+(b-a)\right]$ if $a = 3$ and $b = -2$.

> $\left[a-(b-a)\right]\left[a+(b-a)\right]=\left[2a-b\right]\left[b\right]=\left[6+2\right]\left[-2\right]=-16$.

$\boxed{\dfrac{4}{9}}$

[AMC 12B, #18]

2. Tanea randomly selects a number from the set {1, 2, 3, 4, 5, 6, 7, 8, 9}. Antoine then randomly selects a number (not necessarily distinct) from the same set. Determine exactly the probability that Tanea's number is greater than Antoine's number.

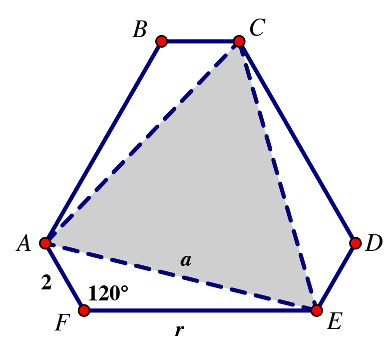
> *Each person has* 9 *numbers to choose from, so there are* $9 \bullet 9 = 81$ *possible combined choices. There are* 9 *choices where Tanea and Antoine choose the same number; of the remaining 72 choices, half of them result in Tanea having the greater number. The probability is* $\dfrac{36}{81}=\dfrac{4}{9}$.

$r = \boxed{6}$

[AMC 12A, #17]

3. Equiangular hexagon $ABCDEF$ has sides $AB = CD = EF = r$ and $BC = DE = FA = 2$. If the area of $\triangle ACE$ is $13\sqrt{3}$, determine exactly the value of r.

> *The interior angles all have measure* 120°*, so by SAS,* $\triangle ABC \cong \triangle CDE \cong \triangle EFA$. *This makes* $\triangle ACE$ *equilateral, so if it has side length a, then* $\dfrac{a^2\sqrt{3}}{4}=13\sqrt{3} \Rightarrow a^2=52$. *Applying the Law of Cosines to* $\triangle EFA$: $52=2^2+r^2-4r\cos 120°$
>
> $\Rightarrow r^2+2r-48=0 \Rightarrow (r+8)(r-6)=0 \Rightarrow r=\cancel{-8}, 6$.



$\boxed{\dfrac{9}{16}}$

[AMC 12A, #16]

4. A frog makes 3 jumps, each exactly 1 meter long. The directions of the jumps are chosen independently at random from among {north, south, east, west}. Determine exactly the probability that the frog's final position is no more than 1 meter from its starting position.

> *The direction of jump #1 is irrelevant, so WLOG, let's assume the frog initially jumps west. Break jump #2 down into cases: (1) if #2 is east, the frog is back to its starting point, and jump #3 must land it 1 meter away... P = 1. (2) if jump #2 is west again, only jump #3 east will work... P = 1/4. (3) if jump #2 is north, only jump #3 east or south will work... P = 1/2. (4) jump #2 south is congruent to case 3... P = 1/2. Overall,* $P = \underbrace{1}_{jump\#1} \cdot \underbrace{\frac{1}{4}}_{jump\#2} \cdot \underbrace{\left(1+\frac{1}{4}+\frac{1}{2}+\frac{1}{2}\right)}_{jump\#3} = \frac{1}{4}\cdot\frac{9}{4}=\frac{9}{16}$.

Minnesota State High School Mathematics League

2010-11 Meet 5, Team Event

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

_______________ 1. The rules of a particular board game state that if a playing piece is at the point (x, y) on the coordinate plane, then it is allowed to move to either $(x+1,\ y)$ or $(x,\ y+1)$.
If the playing piece starts at (0, 0) and moves randomly, ending up at (4, 4), calculate the probability that it will trace two entire sides of the square $1 \le x \le 3,\ 1 \le y \le 3$ as it moves.

_______________ 2. Two players play a game using an n x n square grid of dots, with n an odd positive integer. The players alternate turns, where each turn involves drawing a single horizontal or vertical line segment that connects two adjacent dots. The first player to completely enclose any 1-unit square loses. In terms of n, what is the largest number of turns a game can last?

_______________ 3. Logan is constructing a scale model of his town. The town's water tower stands 125 feet high, the top portion being a sphere that holds 32,000 gallons of water. If Logan's miniature water tower is 15 inches high, how many gallons of water should it hold?

_______________ 4. A ten-inch-long candy cane falls to the ground and randomly breaks (cleanly) at two independent places. Calculate the probability that none of the three newly-created small pieces is more than five inches long.

$(m,n)=$ _______________ 5. Jon and Danielle play a game starting with 11 sticks. They take turns, removing either 1, 2, or 3 sticks on each turn. Whoever removes the last stick loses. Danielle goes first, leaving m sticks for Jon. After Danielle's second turn, n sticks are left, and after her third turn, Jon loses. Determine the ordered pair (m, n).

_______________ 6. In *Figure 6*, equilateral triangles ABC and CDE are constructed on $\overleftrightarrow{AE}$. $\overleftrightarrow{BD}$ and $\overleftrightarrow{AE}$ meet at F, as shown. If ABC and CDE have areas 4 and 9, respectively, calculate the area of $\triangle ABF$.

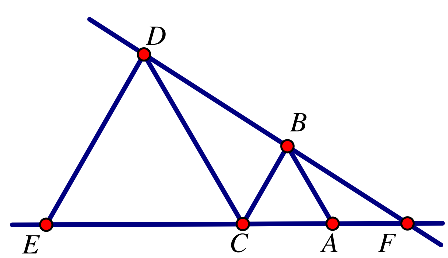


Figure 6

Team: _______________________

Minnesota State High School Mathematics League

2010-11 Meet 5, Team Event

SOLUTIONS (page 1)

$\boxed{\dfrac{4}{35}}$ ***or*** $\boxed{\approx 0.114}$ ***or*** $\boxed{\approx 11.4\%}$

1. The rules of a particular board game state that if a playing piece is at the point (x, y) on the coordinate plane, then it is allowed to move to either $(x + 1,\ y)$ or $(x,\ y + 1)$. If the playing piece starts at (0, 0) and moves randomly, ending up at (4, 4), calculate the probability that it will trace two entire sides of the square $1 \le x \le 3,\ 1 \le y \le 3$ as it moves.

$\boxed{\dfrac{3n^2 - 2n + 1}{2}}$

2. Two players play a game using an n x n square grid of dots, with n an odd positive integer. The players alternate turns, where each turn involves drawing a single horizontal or vertical line segment that connects two adjacent dots. The first player to completely enclose any 1-unit square loses. In terms of n, what is the <u>largest</u> number of turns a game can last?

$\boxed{\dfrac{4}{125}}$ ***or*** $\boxed{0.032}$

3. Logan is constructing a scale model of his town. The town's water tower stands 125 feet high, the top portion being a sphere that holds 32,000 gallons of water. If Logan's miniature water tower is 15 inches high, how many gallons of water should it hold?

[AMC 12A, #7]

$\boxed{\dfrac{1}{4}}$ ***or*** $\boxed{0.25}$ ***or*** $\boxed{25\%}$

4. A ten-inch-long candy cane falls to the ground and randomly breaks (cleanly) at two independent places. Calculate the probability that none of the three newly-created small pieces is more than five inches long.

[<u>Mathematics Teacher</u>, Oct. 2004]

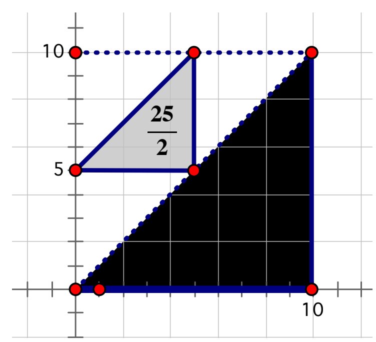


Figure 4

$(m,n) = \boxed{(9,5)}$

5. Jon and Danielle play a game starting with 11 sticks. They take turns, removing either 1, 2, or 3 sticks on each turn. Whoever removes the last stick loses. Danielle goes first, leaving m sticks for Jon. After Danielle's second turn, n sticks are left, and after her third turn, Jon loses. Determine the ordered pair (m, n).

$\boxed{8}$

6. In *Figure 6*, equilateral triangles ABC and CDE are constructed on $\overleftrightarrow{AE}$. $\overleftrightarrow{BD}$ and $\overleftrightarrow{AE}$ meet at F, as shown. If ABC and CDE have areas 4 and 9, respectively, calculate the area of $\triangle ABF$.

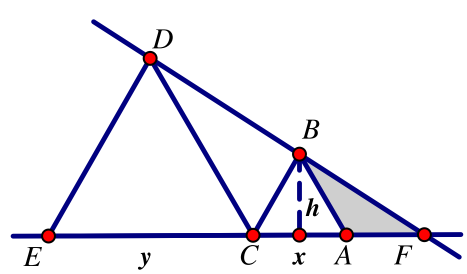


Figure 6

Minnesota State High School Mathematics League

2010-11 Meet 5, Team Event

SOLUTIONS (page 2)

1. *Essentially, we want the game piece to pass through both* (1, 1) *and* (3, 3) *on its way to* (4, 4). *Furthermore, there are only two ways to get from* (1, 1) *to* (3, 3) *while tracing two sides of the square: {up, up, right, right} and {right, right, up, up}. There are, then,* 2 *ways to get from* (0, 0) *to* (1, 1); 2 *ways to get from* (1, 1) *to* (3, 3); *and* 2 *ways to get from* (3, 3) *to* (4, 4). *Multiplying, we have* 8 *legal paths out of a total of* $\binom{8}{4} = 70$ *(using reasoning from Event C).* $\frac{8}{70} = \boxed{\frac{4}{35}}$.

2. *Using the logic from Event A #2, we first count the number of possible 1-unit line segments, then subtract interior segments (each of which can "undo" at most two squares). It takes n–1 horizontal segments to connect the n points in each of the n rows, for a total of* $n(n-1)$ *segments. The same argument can be made for the n columns. Overall, we need* $2n(n-1)$ *segments. Now, note that an n x n grid describes* $(n-1)(n-1)$ *one-unit squares, an even number. To "undo" these will require the removal of half that many interior segments, leaving* $2n(n-1) - \frac{1}{2}(n-1)(n-1) = \frac{4n(n-1)}{2} - \frac{(n-1)(n-1)}{2}$ $= \frac{(n-1)(4n-(n-1))}{2} = \frac{(n-1)(3n+1)}{2} = \frac{3n^2-2n-1}{2}$ *segments. The next segment drawn loses, so add* 1: $\boxed{\frac{3n^2-2n+1}{2}}$.

3. *Let R = radius of sphere on water tower; r = radius of model. The scale factor is* $\frac{r}{R} = \frac{1.25\ feet}{125\ feet} = \frac{1}{100}$, *so the ratio of volumes is* $\left(\frac{1}{100}\right)^3 = \frac{1}{1{,}000{,}000}$. *Since gallons are units of volume, we divide* 32,000 *by* 1,000,000 *to get* $\boxed{0.032}$.

4. *Imagine the candy cane laid on a number line, with endpoints at* 0 *and* 10. *Let x be the location of the first break, and y the location of the second break (with y > x). Certainly* $0 \le x \le 5$ *and* $5 \le y \le 10$. *Furthermore, y cannot lie more than 5 units to the right of x, so:* $y \le x + 5$. *We can graph these relationships as a system of inequalities! (See Figure 4 on the previous page.) The solution region is a triangle with area 25/2; the guidelines presented by the problem* ($0 \le x \le 10$, $0 \le y \le 10$, $y > x$) *form a triangle with area 50. The probability of achieving the desired conditions is 25/2 divided by 50, or* $\boxed{1/4}$.

5. *Work backwards from 1 stick. If Jon leaves either 2, 3, or 4 sticks for Danielle, she can take all but 1 stick. Danielle can force this situation by leaving* ***5*** *sticks for Jon on the previous turn, which means he left 6, 7, or 8 sticks for her. Thus, she must remove 2 sticks initially, leaving* ***9****. Therefore, (m, n) =* $\boxed{(9,5)}$.

6. *Let AC = x, CE = y. Then* $\left(\frac{x}{y}\right)^2 = \frac{4}{9}$, *so* $\frac{x}{y} = \sqrt{\frac{4}{9}} = \frac{2}{3}$. *Since* $\triangle ABF \sim \triangle CDF$, $\frac{AF}{AF+x} = \frac{2}{3} \Rightarrow 2 \cdot AF + 2x = 3 \cdot AF$ $\Rightarrow AF = 2x$. *Also,* $Area[\triangle ABC] = \frac{1}{2}xh = 4$, *so* $h = \frac{8}{x}$. *Note that* $\triangle ABF$ *and* $\triangle ABC$ *share the same height, h.* *Thus,* $Area[\triangle ABF] = \frac{1}{2} \cdot AF \cdot h = \frac{1}{2}(2\cancel{x})\left(\frac{8}{\cancel{x}}\right) = \boxed{8}$.