Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

$x =$ ________________ 1. If $\dfrac{1}{\left(\dfrac{1}{\left(\frac{1}{x}\right)}\right)} = \dfrac{2}{3}$, determine exactly the value of x.

________________ 2. The LCM of 123, 231, and 312 can be written as a power of 2 multiplied by five other prime numbers. Do so.

________________ 3. I'm out for lunch at my favorite café, but I only have \$15.00. If the soup-and-sandwich combo I want to order costs \$13.00, and sales tax is 7%, what is the <u>minimum</u> whole-number percent-off discount coupon I must hold in my wallet to allow me to still leave an 18% tip?
(Note: tax and tip are applied after the coupon, but not to each other.)

________________ 4. A certain positive integer is three greater than a multiple of 5, five greater than a multiple of 8, and eight greater than a multiple of 13. Determine the value of the <u>least</u> such integer.

Name: ______________________________ Team: ______________________________

Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event A

SOLUTIONS

$x = \boxed{\frac{3}{2}}$

1. If $\dfrac{1}{\left(\dfrac{1}{\left(\frac{1}{x}\right)}\right)} = \dfrac{2}{3}$, determine exactly the value of x.

The left side simply asks us to take the reciprocal of x three times. Two of those reciprocals "undo" each other, so we're left with $\frac{1}{x} = \frac{2}{3}$, *and* $x = \frac{3}{2}$.

$\boxed{2^3 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \cdot 41}$

2. The LCM of 123, 231, and 312 can be written as a power of 2 multiplied by five other prime numbers. Do so.

Since all three numbers have digits that sum to a multiple of 3, each number must be itself a multiple of 3:

$123 = 3 \cdot 41$ $\quad$ $231 = 3 \cdot 7 \cdot 11$ $\quad$ $312 = 2^3 \cdot 3 \cdot 13$

So LCM (123, 231, 312) = $2^3 \cdot 3 \cdot 7 \cdot 11 \cdot 13 \cdot 41$.

$\boxed{\text{8\% off}}$

(accept "8")

3. I'm out for lunch at my favorite café, but I only have \$15.00. If the soup-and-sandwich combo I want to order costs \$13.00, and sales tax is 7%, what is the <u>minimum</u> whole-number percent-off discount coupon I must hold in my wallet to allow me to still leave an 18% tip?
(Note: tax and tip are applied after the coupon, but not to each other.)

Tax and tip together comprise 25% of the food order, so the subtotal before tax & tip must be no more than $\frac{\$15.00}{125\%} = \frac{\$15.00}{1.25} = \$12.00$.

Amount of discount = \$13.00 – \$12.00 = \$1.00 $\Rightarrow$ $13\overline{)1.000...} = .076...$, -91, 90 $\Rightarrow$ *8% off.*

$\boxed{333}$

4. A certain positive integer is three greater than a multiple of 5, five greater than a multiple of 8, and eight greater than a multiple of 13. Determine the value of the <u>least</u> such integer.

There are many trial and error methods for solving this problem, but here we present a very elegant and instructive solution:
Let x be the integer in question. Since x is three greater than a multiple of 5, set x equal to 5k + 3. Using modular arithmetic, $x \equiv 5 \bmod 8$ *and* $x \equiv 8 \bmod 13$. *Substituting,*

$$5k + 3 \equiv 5 \bmod 8 \Rightarrow 5k \equiv 2 \bmod 8 \Rightarrow k \equiv 2 \bmod 8;$$

$$5k + 3 \equiv 8 \bmod 13 \Rightarrow 5k \equiv 5 \bmod 13 \Rightarrow k \equiv 1 \bmod 13.$$

By the first result, $k \in \{2, 10, 18, 26, ...\}$ *By the second result,* $k \in \{1, 14, 27, 40, ...\}$

The first k satisfying both conditions is k = 66, making x = 5k + 3 = 5(66) + 3 = 333.

(For more information on problems like these, see:
http://en.wikipedia.org/wiki/Modular_arithmetic
http://en.wikipedia.org/wiki/Chinese_remainder_theorem)

Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

__________ 1. *Figure 1* shows the obtuse angle between lines ℓ_1 and ℓ_2 as 133°, and the obtuse angle between lines ℓ_1 and ℓ_3 as 144°. Determine the measure, in degrees, of the obtuse angle between lines ℓ_2 and ℓ_3.

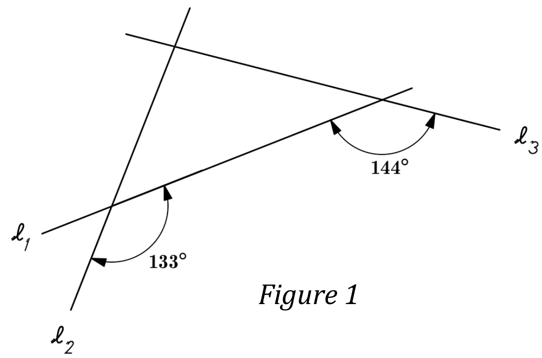


Figure 1

$m\angle BDE =$ __________ 2. In right triangle *ABC (Figure 2)*, point *D* is on side $\overline{AC}$ and *E* is on $\overline{AB}$. Given $m\angle ACB = m\angle ABD = m\angle ADE = 37°$, determine the degree-measure of $\angle BDE$.

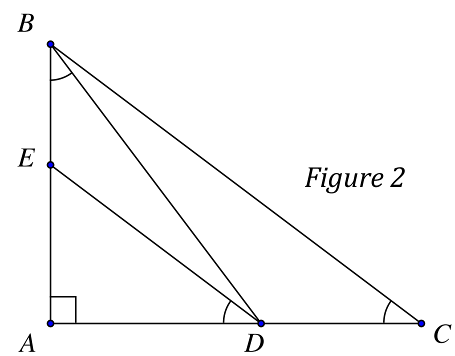


Figure 2

$m\angle ABC =$ __________ 3. In pentagon *PENTA*, the distances *PE*, *PN*, *PT*, *PA*, *ET* and *NA* are all equal *(Figure 3)*. If $m\angle EPA = 76°$, determine the degree-measure of $\angle PEN$.

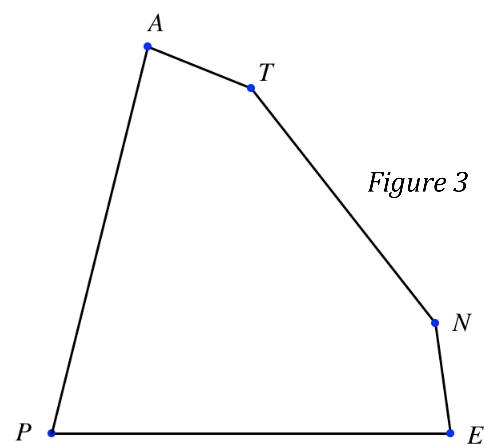


Figure 3

__________ 4. Some angles in a convex 21-gon are right angles, and the rest are congruent obtuse angles. What is the largest possible degree-measure for an angle in this polygon?

Name: ________________ Team: ________________

Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event B

SOLUTIONS

97°

1. *Figure 1* shows the obtuse angle between lines ℓ_1 and ℓ_2 as 133°, and the obtuse angle between lines ℓ_1 and ℓ_3 as 144°. Determine the measure, in degrees, of the obtuse angle between lines ℓ_2 and ℓ_3.

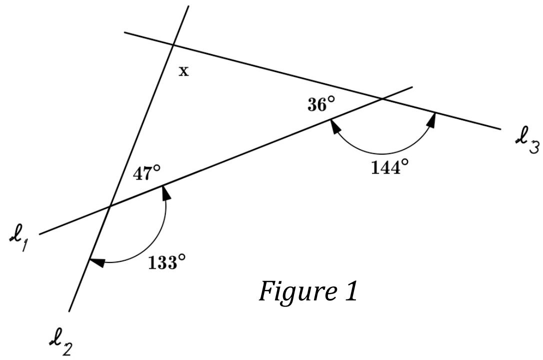


Figure 1

> *The interior angles shown are the supplements of the given obtuse angles. Using the triangle enclosed by the three lines,* $47°+36°+x=180°$ *implies that* $x=97°$.

$m\angle BDE =$ **16°**

2. In right triangle ABC *(Figure 2)*, point D is on side $\overline{AB}$ and E is on $\overline{AC}$.
Given $m\angle ACB = m\angle ABD = m\angle ADE = 37°$, compute the degree-measure of $\angle BDE$.

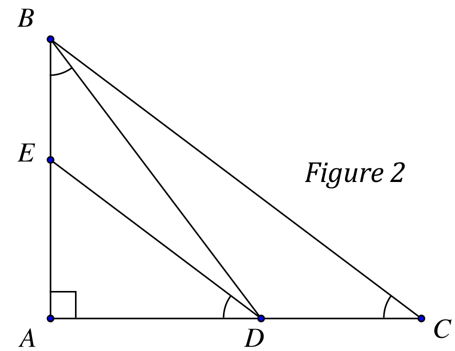


Figure 2

> *Within* $\triangle ABC$, $\angle ABC$ *and* $\angle C$ *are complements, so* $m\angle ABC = 90° - 37° = 53°$. $\angle CBD$ *and* $\angle BDE$ *are alternate interior angles, and since* $\overline{BC} \parallel \overline{DE}$, $\angle CBD$ *and* $\angle BDE$ *are congruent.* $m\angle BDE = 53° - 37° = 16°$.

$m\angle ABC =$ **82°**

3. In pentagon $PENTA$, the distances PE, PN, PT, PA, ET and NA are all equal *(Figure 3)*. If $m\angle EPA = 76°$, determine the degree-measure of $\angle PEN$.

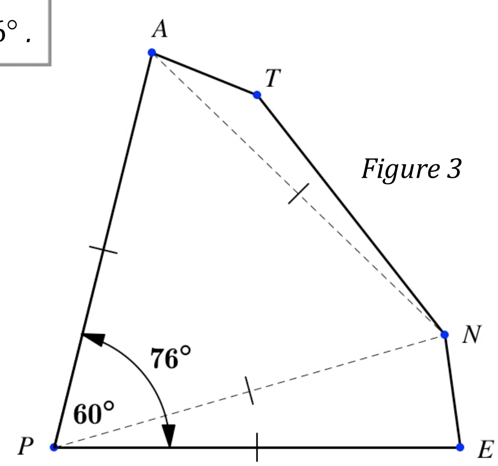


Figure 3

> $\triangle APN$ *is equilateral, so* $m\angle APN = 60°$. *Thus* $m\angle NPE = 76° - 60° = 16°$. *Since* $\triangle PEN$ *is isosceles,* $m\angle PEN = \dfrac{180° - 16°}{2} = 82°$.

175°

4. Some angles in a convex 21-gon are right angles, and the rest are congruent obtuse angles. What is the <u>largest</u> possible degree-measure for an angle in this polygon?

> *Let r = the number of right angles. Then there are (21 – r) obtuse angles, of a measure we'll call m. The sum of the angles in the 21-gon is then* $180°(21-2) = 90°r + m(21-r)$.
>
> *Rewriting the left side as* $90°\cdot 38$, *we have* $(21-r)m = 90°(38-r) \Rightarrow m = 90°\left(\dfrac{38-r}{21-r}\right)$.
>
> *We want* $\dfrac{38-r}{21-r} < 2$, *occurring when r < 4. The nearest value is r = 3, leading to* $m = 175°$.

Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

$\cos\theta =$ ____________ 1. Using *Figure 1*, showing certain segments labeled as length 1, determine exactly the value of $\cos\theta$.

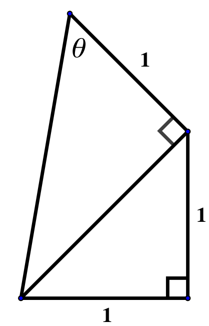


Figure 1

$\frac{\tan x}{\cot x} =$ ____________ 2. Given that the ratio of cos x to sin x is 3:2, determine exactly the ratio (tan x):(cot x).

$\tan(180° - \beta) =$

____________ 3. If $\cos\alpha = \frac{1}{3}$ and α and β are acute complements, determine exactly $\tan(180° - \beta)$.

____________ 4. Determine exactly the coordinates (both of them) of the highest point on the graph of $f(x) = \left[2\cos\left(3x - \frac{\pi}{4}\right) - 5\right]^2$ on the interval $0 \le x \le 2$.

Name: ______________________ Team: ______________________

Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event C

SOLUTIONS

$\cos\theta = \boxed{\dfrac{\sqrt{3}}{3}}$

(accept $\dfrac{1}{\sqrt{3}}$ here as a kind of grace period, but warn students about the requirements of "determine exactly")

1. Using *Figure 1*, showing certain segments labeled as length 1, determine exactly the value of cos *θ.*

The bottom-right triangle is right isosceles, so its hypotenuse has length $\sqrt{2}$. Using the Pythagorean Theorem on the upper triangle yields a hypotenuse of length $\sqrt{3}$, so the desired cosine is $\dfrac{adj}{hyp} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$.

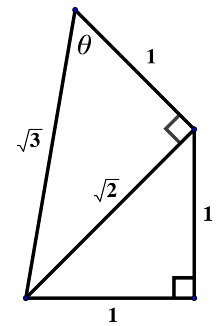


Figure 1

$\dfrac{\tan x}{\cot x} = \boxed{\dfrac{4}{9}}$

2. Given that the ratio of cos *x* to sin *x* is 3:2, determine exactly the ratio (tan *x*):(cot *x*).

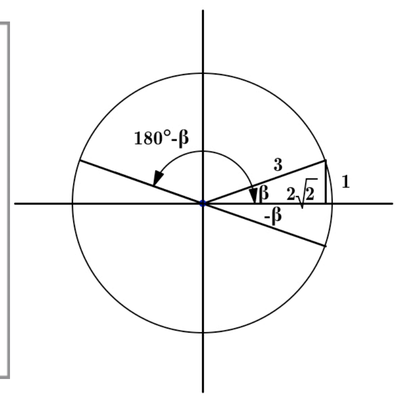
$\dfrac{\cos x}{\sin x} = \cot x = \dfrac{3}{2}$, *so* $\tan x = \dfrac{2}{3}$, *and* $\dfrac{\tan x}{\cot x} = \dfrac{\left(\frac{2}{3}\right)}{\left(\frac{3}{2}\right)} = \dfrac{2}{3}\cdot\dfrac{2}{3} = \dfrac{4}{9}$.

$\tan\left(180° - \beta\right) = \boxed{\dfrac{-\sqrt{2}}{4}}$

(As in #1, accept equivalents such as $\dfrac{-1}{2\sqrt{2}}$, but be instructive to students about the new grading guidelines)

3. If $\cos\alpha = \dfrac{1}{3}$ and $\alpha + \beta = 90°$, determine exactly $\tan\left(180° - \beta\right)$.

Since α and β are complements, their cofunctions are equal, so $\sin\beta = \dfrac{1}{3}$. The circle diagram shown to the right reveals that $\tan\beta = \dfrac{1}{2\sqrt{2}}$, and via reflections and negative-angle identities, we see that $\tan\left(180° - \beta\right) = \tan\left(-\beta\right) = -\tan\beta = \dfrac{-1}{2\sqrt{2}} = \dfrac{-\sqrt{2}}{4}$.



$\boxed{\left(\dfrac{5\pi}{12}, 49\right)}$

(Award 1 point for each correct coordinate)

4. Determine exactly the coordinates (both of them) of the highest point on the graph of $f\left(x\right) = \left[2\cos\left(3x - \dfrac{\pi}{4}\right) - 5\right]^2$ on the interval 0 ≤ *x* ≤ 2.

Thinking transformationally, the range of y = cos x is −1 ≤ cos x ≤ 1, so we:

- *Multiply all sides by 2: −2 ≤ 2 cos x ≤ 2*
- *Subtract 5 from all sides: −7 ≤ 2 cos x − 5 ≤ −3*
- *Square all sides, which reverses the inequality: 9 ≤ (2 cos x − 5)^2 ≤ 49*

So the maximum possible value occurs at 49 - but is there an x-value between 0 and 2 that pairs with 49?

$\left[2\cos\left(3x - \dfrac{\pi}{4}\right) - 5\right]^2 = 49 \Rightarrow 2\cos\left(3x - \dfrac{\pi}{4}\right) - 5 = \pm 7 \Rightarrow \cos\left(3x - \dfrac{\pi}{4}\right) = -1$ *or* ~~6~~

$\Rightarrow 3x - \frac{\pi}{4} = \pi + 2k\pi \Rightarrow 3x = \frac{5\pi}{4} + 2k\pi \Rightarrow x = \frac{5\pi}{12} + \frac{8\pi}{12}k$, *and the only x that works is* $\frac{5\pi}{12}$.

Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

____________ 1. Determine exactly the <u>least</u> value of x that satisfies the equation $(x-4)(x+4)=9$.

____________ 2. Determine exactly the coordinates (both of them) of the highest point on the graph of $y+x^2+6x=4$.

k = ____________ 3. For what value of k will the graphs of $y=k$ and $y=-\frac{1}{4}x^2+\frac{3}{2}x-\frac{1}{2}$ intersect at only one point?

____________ 4. Determine exactly each of the four complex roots of $x^2+\frac{25}{x^2}+2x+\frac{10}{x}+2=0$.

Name: ____________________ Team: ____________________

Minnesota State High School Mathematics League

2010-11 Meet 1, Individual Event D

SOLUTIONS

$\boxed{-5}$

1. Determine exactly the <u>least</u> value of x that satisfies the equation $(x-4)(x+4)=9$.

The left side of the equation simplifies into the difference of two squares: $x^2-16=9 \Rightarrow x^2=25 \Rightarrow x=\pm 5$. *The lesser of those two solutions is –5.*

$\boxed{(-3, 13)}$

(Award 1 point for each correct coordinate)

2. Determine exactly the coordinates (both of them) of the highest point on the graph of $y+x^2+6x=4$.

Completing the square with respect to x on the left side of the equation yields $y+(x+3)^2-9=4$, *and isolating y, we have* $y=-(x+3)^2+13$, *which has its vertex (maximum point) at (–3, 13).*

$k = \boxed{\frac{7}{4}}$

(Also accept 1.75 ***or*** $1\frac{3}{4}$ ***)***

3. For what value of k will the graphs of $y = k$ and $y=-\frac{1}{4}x^2+\frac{3}{2}x-\frac{1}{2}$ intersect at only one point?

This problem is really an exercise in recognition. We are given a horizontal line and a vertically-oriented parabola, which will have a single intersection only at the vertex. The y-coordinate of the vertex can be found, among other methods, by calculating:

$$-\left(\frac{b^2}{4a}+c\right)=-\left(\frac{\left(\frac{3}{2}\right)^2}{4\left(-\frac{1}{4}\right)}+\frac{1}{2}\right)=-\left(-\frac{9}{4}+\frac{2}{4}\right)=\frac{7}{4}.$$

$\boxed{-2\pm i,\ 1\pm 2i}$

(Award 1 point for each pair of correct roots)

4. Determine exactly each of the four complex roots of $x^2+\frac{25}{x^2}+2x+\frac{10}{x}+2=0$.

The two fractional terms are both related to $\frac{5}{x}$, *so rewrite the first two terms as* $\left(x+\frac{5}{x}\right)^2-10$. *Then the entire left side can be expressed as* $\left(x+\frac{5}{x}\right)^2+2\left(x+\frac{5}{x}\right)-8$, *and setting* $y=x+\frac{5}{x}$, *we have* $y^2+2y-8=0 \Rightarrow (y+4)(y-2)=0 \Rightarrow y=-4$ *or* 2.

So $x+\frac{5}{x}=-4 \Rightarrow x^2+4x+5=0 \Rightarrow x=\frac{-4\pm\sqrt{16-20}}{2}=-2\pm i$,

or $x+\frac{5}{x}=2 \Rightarrow x^2-2x+5=0 \Rightarrow x=\frac{2\pm\sqrt{4-20}}{2}=1\pm 2i$.

Minnesota State High School Mathematics League

2010-11 Meet 1, Team Event

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

$n =$ ______________

1. Convex polygon $A_1A_2...A_{2n+1}$ has n congruent angles ($\angle A_{2n+1}A_1A_2 \cong \angle A_1A_2A_3 \cong ... \cong \angle A_{n-1}A_nA_{n+1}$), and the other n + 1 angles are congruent to each other. If $\overline{A_{2n+1}A_1}$ is parallel to $\overline{A_nA_{n+1}}$ and $m\angle A_1A_2A_3 = m\angle A_{2n}A_{2n+1}A_1 - 2°$, find the value of n.

$b =$ ______________

2. Determine the smallest base $b > 10$ in which 2010_b is equivalent to a base-10 multiple of 2010.

$k =$ ______________

3. Calculate the values of k for which the graphs of $y = x$ and $y = -\frac{1}{4}x^2 + kx - \frac{1}{2}$ intersect at only one point.

____

4. Hexagon *HEXGON* can be dissected into right triangles, as shown in *Figure 4*. The nine line segments in the diagram all have different integer lengths, and $\overline{GO}$ has as short a length as possible. Calculate the smallest possible value for the perimeter of *HEXGON*.

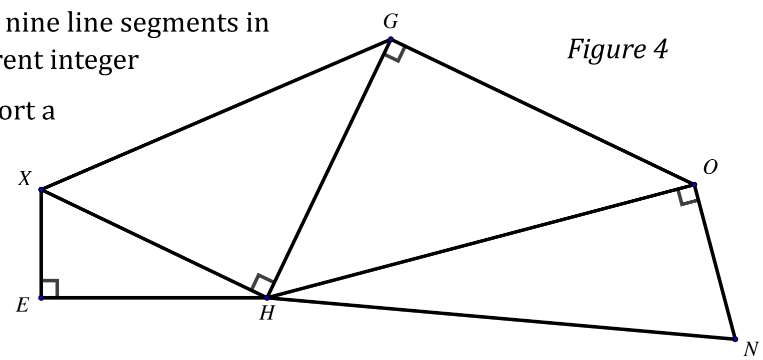


Figure 4

____

5. Determine the smallest possible LCM of $\underline{a}\,\underline{b}\,\underline{c}$, $\underline{b}\,\underline{c}\,\underline{a}$, and $\underline{c}\,\underline{a}\,\underline{b}$ if a, b, and c are distinct digits (with $a < b < c$) and are also the roots of $x^3 + 18x^2 + mx + n = 0$.

$f(x) =$ ______________

6. The graph of $y = f(x)$, $x \le 9$ is the left half of the parabola which intersects the y-axis at –4 and the x-axis at –2. Write a formula for $f(x)$.

Team: ______________

Minnesota State High School Mathematics League

2010-11 Meet 1, Team Event

SOLUTIONS (page 1)

$n = \boxed{9}$

1. Convex polygon $A_1A_2...A_{2n+1}$ has n congruent angles ($\angle A_{2n+1}A_1A_2 \cong \angle A_1A_2A_3 \cong ... \cong \angle A_{n-1}A_nA_{n+1}$), and the other $n + 1$ angles are congruent to each other. If $\overline{A_{2n+1}A_1}$ is parallel to $\overline{A_nA_{n+1}}$ and $m\angle A_1A_2A_3 = m\angle A_{2n}A_{2n+1}A_1 - 2°$, find the value of n.

$b = \boxed{670}$

2. Determine the smallest base $b > 10$ in which 2010_b is equivalent to a base-10 multiple of 2010.

$k = \boxed{\dfrac{2 \pm \sqrt{2}}{2}}$

3. Calculate the values of k for which the graphs of $y = x$ and $y = -\frac{1}{4}x^2 + kx - \frac{1}{2}$ intersect at only one point.

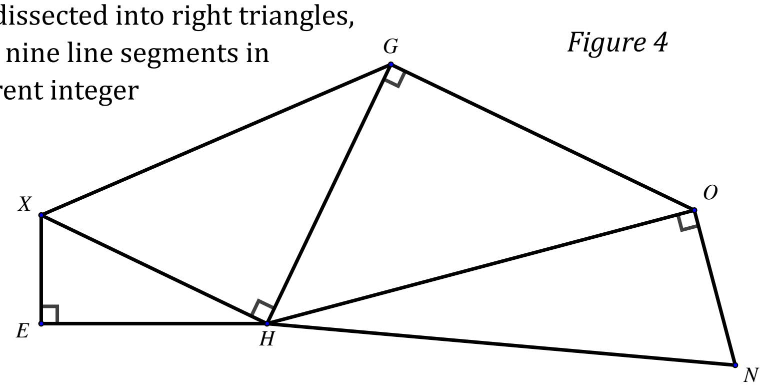
(Award 2 points for each correct value; accept $1 \pm \frac{\sqrt{2}}{2}$***, decimal equivalents, etc.)***

$\boxed{94}$

(Note that the word "calculate" will sometimes be used in cases where a 3-place decimal approximation is not necessary, in order to mask the nature of a solution.)

4. Hexagon *HEXGON* can be dissected into right triangles, as shown in *Figure 4*. The nine line segments in the diagram all have different integer lengths, and $\overline{GO}$ has the shortest length of all those segments. Calculate the <u>smallest</u> possible value for the perimeter of *HEXGON*.

Figure 4



$\boxed{56700}$

5. Determine the smallest possible LCM of $\underline{a}\ \underline{b}\ \underline{c}$, $\underline{b}\ \underline{c}\ \underline{a}$, and $\underline{c}\ \underline{a}\ \underline{b}$ if a, b, and c are distinct digits (with $a < b < c$) and are also the roots of $x^3 + 18x^2 + mx + n = 0$.

$f(x) =$

$\boxed{\frac{1}{10}(x-9)^2 - \frac{121}{10}}$

or...

$\boxed{\frac{1}{10}x^2 - \frac{9}{5}x - 4}$

6. The graph of $y = f(x)$, $x \le 9$ is the left half of the parabola which intersects the y-axis at −4 and the x-axis at −2. Write a formula for $f(x)$.

Minnesota State High School Mathematics League

2010-11 Meet 1, Team Event

SOLUTIONS (page 2)

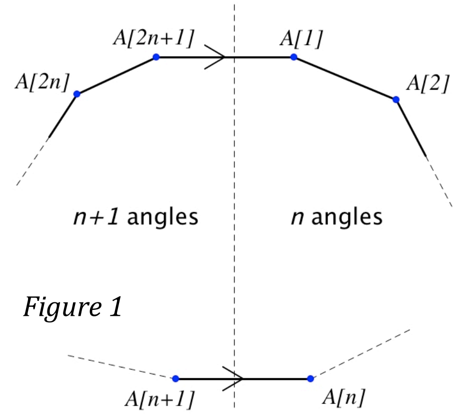


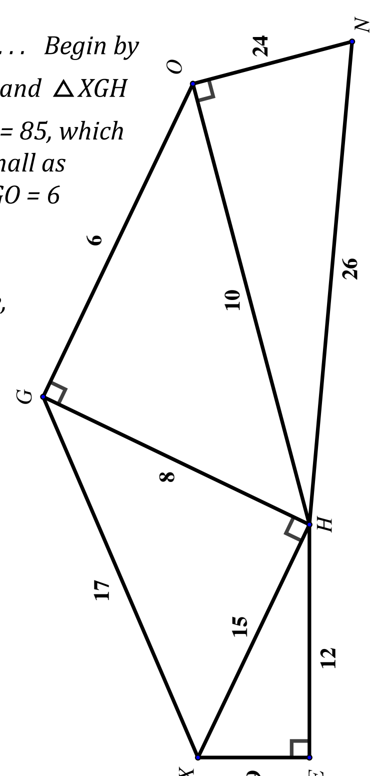
Figure 1

1. *Imagine a line that divides the (2n + 1)-gon in "half", with the n congruent angles of one measure on one side of the line, and the (n +1) congruent angles of another measure on the other side. For the two indicated sides to be parallel, the exterior angles on each side of the dividing line must sum to 180°. Furthermore, if the two types of interior angles differ by 2°, then so do their exterior angles. Set up the equation* $\frac{180°}{n} = \frac{180°}{n+1} + 2°$ *and solve for n to obtain n =* $\boxed{9}$.

2. $2010_b = \underset{b^3}{\underline{2}}\underset{b^2}{\underline{0}}\underset{b}{\underline{1}}\underset{1}{\underline{0}} = b\left(2b^2+1\right)$*, which we are told equals* $2010k = 2\cdot 3\cdot 5\cdot 67\cdot k$ *for some integer* k. *A short investigation of 0, 1, 2, . . . , 9 reveals that if the last digit of* $b\left(2b^2+1\right)$ *is 0, then the last digit of b must also be 0. Set b = 10n for some integer n. Then* $b\left(2b^2+1\right) = \cancel{10}n\left(200n^2+1\right) = \cancel{2}\cdot 3\cdot \cancel{5}\cdot 67\cdot k$. *So 67 must divide either n or* $200n^2+1$. *We want the smallest base b > 10, so let n = 67, which makes b =* $\boxed{670}$.

3. *Substitute x for y and rearrange:* $-\frac{1}{4}x^2 + (k-1)x - \frac{1}{2} = 0$. *By the quadratic formula,* $x = \frac{-(k-1) \pm \sqrt{(k-1)^2 - 4\left(-\frac{1}{4}\right)\left(-\frac{1}{2}\right)}}{2\left(-\frac{1}{4}\right)}$.

A discriminant of 0 leads to a single x, so $(k-1)^2 - 4\left(-\frac{1}{4}\right)\left(-\frac{1}{2}\right) = 0 \Rightarrow (k-1)^2 = \frac{1}{2} \Rightarrow k-1 = \pm\sqrt{\frac{1}{2}} \Rightarrow k = \boxed{\frac{2\pm\sqrt{2}}{2}}$.



4. *The smallest Pythagorean triples are (3, 4, 5), (5, 12, 13), (6, 8, 10), (7, 24, 25), (8, 15, 17), . . . Begin by choosing the smallest value in each triple as the length of* $\overline{GO}$. *GO = 3 fails because GH = 4, and* $\triangle XGH$ *cannot be another 3-4-5 triangle. GO = 5 works, but then HO = 13, forcing ON = 84 and HN = 85, which will make for a very large hexagon perimeter. The key will be to make hypotenuse HO as small as possible so that* $\triangle HON$ *uses a "small" Pythagorean triple. This is accomplished by setting GO = 6 and labeling the diagram as shown. The perimeter of the resulting hexagon is* $\boxed{94}$.

5. *The given cubic polynomial tells us that the sum of the roots, a + b + c, equals 18. Therefore,* $\underline{a}\,\underline{b}\,\underline{c} = 100a + 10b + c = 99a + 9b + (a+b+c) = 99a + 9b + 18 = 9(11a+b+2)$. *Similarly,* $\underline{b}\,\underline{c}\,\underline{a} = 9(11b+c+2)$*, and* $\underline{c}\,\underline{a}\,\underline{b} = 9(11c+a+2)$. *So the LCM contains a 9. Build a table:*

a	b	c	$11a+b+2$	$11b+c+2$	$11c+a+2$	LCM
5	6	7	$63 = 3^2\cdot 7$	$75 = 3\cdot 5^2$	$84 = 2^2\cdot 3\cdot 7$	$2^2\cdot 3^4\cdot 5^2\cdot 7 = 56700$
4	6	8	$52 = 2^2\cdot 13$	$76 = 2^2\cdot 19$	$94 = 2\cdot 47$	$2^2\cdot 3^2\cdot 13\cdot 19\cdot 47 = 417924$
3	6	9	41	$77 = 7\cdot 11$	$104 = 2^3\cdot 13$	$2^3\cdot 3^2\cdot 7\cdot 11\cdot 13\cdot 41 = 2954952$
4	5	9	$51 = 3\cdot 17$	$66 = 2\cdot 3\cdot 11$	$105 = 3\cdot 5\cdot 7$	$2\cdot 3^3\cdot 5\cdot 7\cdot 11\cdot 17 = 353430$
2	7	9	31	$88 = 2^3\cdot 11$	103	$2^3\cdot 3^2\cdot 11\cdot 31\cdot 103 = 2528856$
1	8	9	$21 = 3\cdot 7$	$99 = 3^2\cdot 11$	$102 = 2\cdot 3\cdot 17$	$2\cdot 3^4\cdot 7\cdot 11\cdot 17 = 212058$

The smallest possible LCM is clearly $\boxed{56700}$.

6. *Using vertex form for a quadratic function,* $f(x) = a(x-h)^2 + k \Rightarrow -4 = a(0-9)^2 + k \Rightarrow k = -4 - 81a$.

Rewriting using the x-intercept, $f(x) = a(x-h)^2 + k \Rightarrow 0 = a(-2-9)^2 + (-4-81a) \Rightarrow 4 = 121a - 81a \Rightarrow a = \frac{1}{10}$.

Therefore, $k = -4 - 81a = -4 - \frac{81}{10} = \frac{-121}{10}$*, and one possible equation is* $\boxed{f(x) = \frac{1}{10}(x-9)^2 - \frac{121}{10}}$.