Minnesota State High School Mathematics League

Individual Event

2009-10 Event 5A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. <u>You have an extended 20 minutes</u> for this event.

1. Compute, in simplified form, the value of the 8th expression in this pattern:

   1   2 + 3   4 + 5 + 6   7 + 8 + 9 + 10   ...

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2. In the pattern ..., *a*, 2, *b*, *a*, ..., each number is equal to twice the previous number plus the number two places back. Compute the value of *a*.

***a* =** ______________________________

3. The product of two numbers is equal to their sum. If we add 1 to both numbers, their new product will be five times their original product. What is the original sum?

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4. The product $\underline{3}\underline{4}\underline{5}\cdot\underline{5}\underline{6}\underline{7}$ is not a perfect square, but changing just one of the six underlined digits creates a product of N^2, where N is a positive integer. Compute N.

***N* =** ______________________________

Name ______________________________   Team ______________________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 5B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Calculate the area of the isosceles triangle with side lengths 13, 13, and 24.

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2. If the numerical value of the area of a square plus two times the numerical value of its perimeter is equal to 20, what is the length of one side of the square?

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3. A square with an area of 16 is subdivided into four congruent smaller squares. Calculate the area of the circle that passes through the centers of all four smaller squares.

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4. The base of a right square pyramid is inscribed in the base of a cylinder. The height of the pyramid equals the side length of its base, and is also one-third the height of the cylinder. Find the ratio of the surface area of the pyramid to the surface area of the cylinder.

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Name _____________________________ Team _____________________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 5C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Three standard 6-sided dice are rolled. Express, as the quotient of two relatively prime integers, the probability that the numbers showing on the top faces of the dice sum to 4.

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2. There are 14 applicants for a position. A screening committee is to present to the manager three candidates, listed in alphabetical order to avoid bias. How many such lists are possible?

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3. How many unique license plates can be made using two letters (excluding the letter "O", which looks too much like a zero) and two single-digit numbers? Neither letters nor numbers need to be distinct, and note that plates like A1B0 and 0B1A are both unique.

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4. Evaluate $\dfrac{2^{-n}(2n+1)!}{n!\,[1\cdot 3\cdot 5\cdot \ldots \cdot (2n-1)]}$ for n = 10,000.

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Name __________________________ Team __________________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 5D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. What number is one-fourth of the way from $\frac{2}{5}$ to $\frac{3}{5}$?

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2. An isosceles trapezoid has legs of length 15 and bases of lengths 28 and 52. Find the length of one of the diagonals, given that this length is a prime number.

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3. The first three terms of an infinite geometric sequence are x^2+3x+2, x^2+5x+6, and $x^2+8x+15$, respectively. Compute the value of the term in the sequence which is *closest*, numerically, to 2010.

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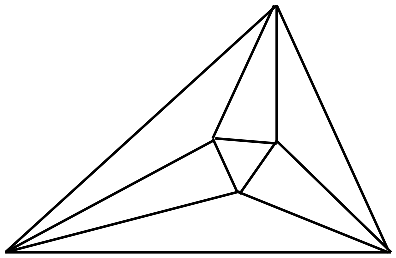
4. A triangle has vertices (0, 0), (0, 3*m*), and (4*m*, 0), and the line $y = mx$ divides the triangle into two triangles of equal perimeter. Find the value of *m*.

***m* =** _____________________________

Name _____________________________ Team _____________________________



Minnesota State High School Mathematics League

Team Event

2009-10 Meet 5

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. Consider a cube of side length 1. The centers of each pair of faces of the cube that share a common edge are connected to form a regular octahedron (a regular polyhedron comprised of 8 congruent equilateral triangular faces). Compute the volume of this octahedron.

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2. The sides of a triangle have lengths 6, 7, and x. What is the largest possible area of such a triangle?

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3. Claire designed a 3 x 3 magic square, where every row, column, and diagonal sums to the same total. Then, to every number in the square, she randomly added or subtracted a number d. The resulting grid is shown in *Figure 3*. Compute d.

109	20	33
18	70	122
59	96	7

Figure 3

***d* =** __________________________

4. In Event A, students were asked to find the value of the 8th expression in the pattern:

   1   2 + 3   4 + 5 + 6   7 + 8 + 9 + 10   ...

   Find the value of the 100th expression in the pattern.

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5. Using distinct digits from the set {0, 1, 2, 3, 4, 7, 8}, how many even integers, greater than 200 but less than 1000, can be written?

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6. One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. By how many square units has the surface area of the rectangular solid decreased?

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Team ______________________________