Minnesota State High School Mathematics League

Individual Event

2009-10 Event 4A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. Compute the value of $\sqrt[3]{3^5+3^5+3^5}$.

2. The expression $(x+y)^3 - x(x+y)^2 - y(x+y)^2$ can be simplified so that it is written as just a single term. Do so.

3. The function f is defined by $f(n) = 3 \cdot f(n-1) - f(n-2)$, where n is any positive integer. If $f(1) = 1$, and $f(2) = \frac{1}{3}$, evaluate $f(7)$.

f (7) =

4. In the equation $\dfrac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}} = \dfrac{\sqrt{x}+\sqrt{y}}{2}$, both x and y are nonnegative integers.

   Compute the sum $x + y$.

x + y =

Name

Team

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 4B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

Questions #1-3 all refer to a nine-sided regular polygon that is labeled A_1 A_2... A_9 and inscribed in a circle of radius 1.

1. Tangents to the circle at A_1 and A_7 meet at point P. How long is $\overline{A_1P}$?

A_1P= ________________________

2. Secants containing $\overline{A_3A_9}$ and $\overline{A_4A_5}$ meet at point Q. What (in degrees) is the measure of $\angle A_5QA_9$?

$m\angle A_5QA_9$ = __________________

3. The secant containing $\overline{A_2A_9}$ meets the extension of the diameter containing A_8 at point R. What (in degrees) is the measure of $\angle A_9RA_8$?

$m\angle A_5RA_8$ = __________________

4. In *Figure 4*, $m\angle ABD = 120°$ and $BC = CD = 1$. A circle is drawn through C and D, tangent to $\overline{AB}$ at T. What will be the length of BT?

BT = ________________________

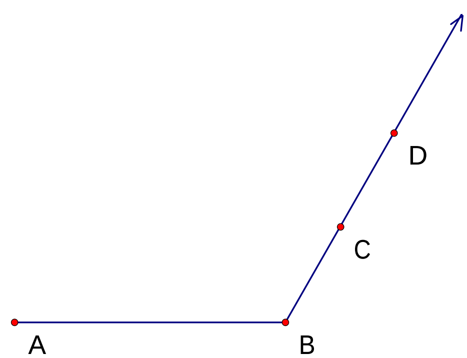

Figure 4

Name ____________________________ Team ____________________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 4C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. What is the 2010^{th} positive odd number?

____

2. Find the sum of the infinite geometric series whose first two terms are 6! and 5! .

____

3. The sum of the first ten terms of an arithmetic sequence is four times the sum of the first five terms. If the first term of the sequence is a_1 and the common difference is d, compute the ratio $a_1 : d$.

a_1 : d = ______________________________

4. If $f(x) = 1 - \dfrac{1}{x}$, find the exact value of x for which $\underbrace{f\Big(f\Big(f\Big(f\Big(f\Big(f\Big(f\ldots\big(f(x)\big)\Big)\Big)\Big)\Big)\Big)\Big)}_{\text{2009 applications of the function } f} = 2010$.

x = ______________________________

Name ______________________________ Team ______________________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 4D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Give the coordinates of the center of the circle described by $x^2 + y^2 - 12x + 10y - 38 = 0$.

(h, k) = ______________________

2. Compute the area of the circle described by $x^2 + y^2 + 2x + 6y + 3 = 0$.

____

3. The asymptotes of a hyperbola are the lines $y = 2x$ and $y = -2x$. If the hyperbola passes through the point (9, 16), compute the x-coordinate of the hyperbola's positive x-intercept.

x = ______________________

4. A hyperbola has its foci on the x-axis and passes through the points (–1, 0), (2, 0), and (–2, 1). Compute the x-coordinate of the right-most focus.

x = ______________________

Name __________________________ Team __________________________

Minnesota State High School Mathematics League

Team Event

2009-10 Meet 4

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. A triangle inscribed in a circle has side lengths 12, $12\sqrt{2}$, and $6\sqrt{6}+6\sqrt{2}$. Compute the length of the circle's diameter.

____

2. What is the least positive integer $n > 1$ for which the expression $\sqrt{1+2+3+...+n}$ simplifies to an integer?

***n* =** ____________________________

3. The expression $\dfrac{\left(\sqrt{3}+\sqrt{5}\right)\left(\sqrt{5}+\sqrt{2}\right)}{\sqrt{2}+\sqrt{3}+\sqrt{5}}$ can be rationalized into a single fraction whose denominator is a positive integer. Do so.

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4. Given $f(\theta)=(1+\cos\theta)\sqrt{\dfrac{\sec\theta-1}{\sec\theta+1}}$, express $f(0)+f\left(\dfrac{\pi}{6}\right)+f\left(\dfrac{\pi}{4}\right)+f\left(\dfrac{\pi}{3}\right)$ accurate to three places to the right of the decimal.

____

5. When the radical equation $\sqrt{x+1-2\sqrt{x}}=15$ is solved using the typical method of squaring both sides repeatedly, two solutions are discovered for *x*, but one of these solutions is extraneous. Find the value of that <u>extraneous</u> solution.

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6. A parabola is defined as the curve containing all points equidistant from a focus *F* and a line called the directrix. Let us define a *quasi-parabola* to be the curve containing all points equidistant from *F* and a <u>line segment</u> called the directrix. Find all *x*- and *y*-intercepts of the quasi-parabola with *F* = (9, 9) and directrix with endpoints *A* = (3, 7) and *B* = (7, 5).

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Team ____________________________