Minnesota State High School Mathematics League

Individual Event

2009-10 Event 3A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. As the value of k changes, the equation $(2x-3y+6)+k(2x+y-10)=0$ describes a family of lines. What value of k identifies the vertical line in this family?

***k* =** ____________________________

2. Solve the system: $\begin{cases} 2x-3y+6=0 \\ 2x+y-10=0 \end{cases}$

***(x, y)* =** ____________________________

3. The system $\begin{cases} p+q+2r=1 \\ 2p-q=3 \\ -p+4r=1 \end{cases}$ has a unique solution. Find it.

***(p, q, r)* =** ____________________________

4. Consider all possible ordered triples of integers (x, y, z) which solve the system $\begin{cases} x+3y-z=1 \\ 3x-y-2z=-1 \end{cases}$. There are two specific single-digit positive integers, n and d, such that all of the x-values in those ordered triples can be written in the form $mn + d$, where m is any integer. Compute the values of n and d.

***n* =** ____________ ***d* =** ____________

Name ____________________________ Team ____________________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 3B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. *Figure 1* shows square *ABCD*, of side length 2, inscribed in parallelogram *AECF*. If $m\angle F = 60°$, compute the length of $\overline{FC}$.

FC = ____________________

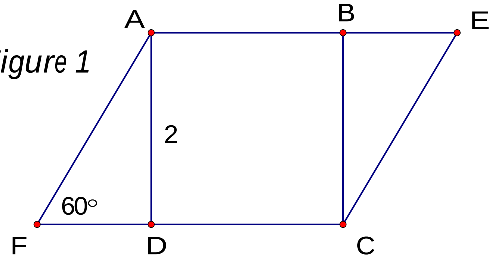

Figure 1

2. *Figure 2* shows square *ABCD*, of side length 2, inscribed in kite *KITE* so that $\triangle DCE$ is equilateral. If angles *K* and *T* are right angles, compute the length of $\overline{ET}$.

ET = ____________________

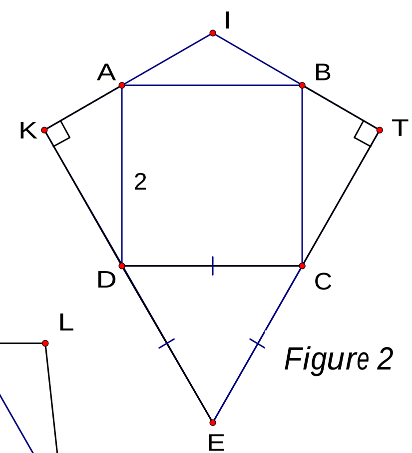

Figure 2

3. *Figure 3* shows square *ABCD*, of side length 2, inscribed in trapezoid *KLMN* $\left(\overline{KL} \parallel \overline{NM}\right)$ so that $\triangle AND$ is equilateral. Compute the length of $\overline{NK}$.

NK = ____________________

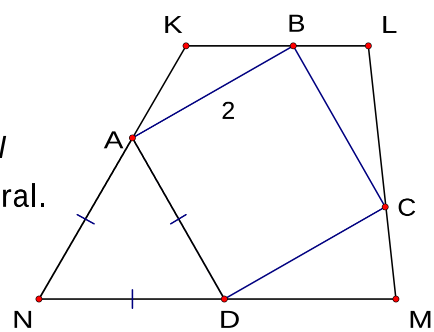

Figure 3

4. A square of side length 2 is inscribed in a regular hexagon so that two sides of the square are parallel to two sides of the hexagon. Find the shortest distance between two of the parallel sides.

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Name ____________________ Team ____________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 3C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Compute the value of $\mathrm{Sin}^{-1}\left(\frac{1}{2}\right)+\mathrm{Cos}^{-1}\left(\frac{-1}{2}\right)$.

____

2. Express the range of the function $f(x)=\cos\left(\mathrm{Tan}^{-1}x\right)$. Do this by placing inequality symbols in the boxes, and real numbers, $-\infty$, or $+\infty$ in the blanks.

_________ ☐ *f (x)* ☐ _________

3. Right triangle *ABC* (shown in *Figure 3*) has legs of lengths *AB* = 4 and *AC* = 6, and point *D* is the midpoint of side $\overline{AC}$. Compute $\sin\angle CBD$.

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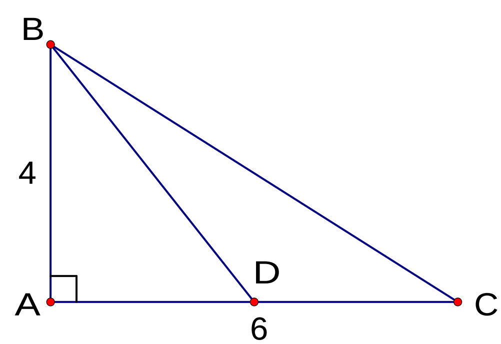

Figure 3

4. The side lengths of a certain triangle are three consecutive integers, and the smallest angle in the triangle has a tangent of $\frac{2\sqrt{6}}{5}$. Find the exact value of the triangle's perimeter.

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Name _____________________________ Team _____________________________

Minnesota State High School Mathematics League

Individual Event

2009-10 Event 3D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. If $\log_4 M = \frac{5}{2}$, then M can be written in the form 2^n. Compute n.

n = ______________________

2. Compute the value of the sum: $\log\frac{1}{2} + \log\frac{2}{3} + \log\frac{3}{4} + \ldots + \log\frac{n}{n+1} + \ldots + \log\frac{99}{100}$.

____

3. If $b = \log_3 x$, find all real values of x which satisfy $\log_b\left(\log_3 x^2\right) = 2$.

x = ______________________

4. If log 80 = a and log 45 = b, write log 6 as a simplified expression involving a and b.

log 6 = ______________________

Name ______________________ Team ______________________

Minnesota State High School Mathematics League

Team Event

2009-10 Meet 3

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. Find the value of n such that $\frac{1}{1+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ldots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}=100$.

n = ____________________

2. The sides of a triangle have lengths $\log_8 a$, $\log_{16} a$, and $\log_{64} a$ for some value of a. Compute the sine of the smallest angle in this triangle.

____

3. *Figure 3* shows square *ABCD*, of side length 2, inscribed in trapezoid *KLMN* so that ΔAND is equilateral. If $m\angle M=\alpha$, express the length of $\overline{LM}$ in terms of $\sin\alpha$.

LM = ____________________

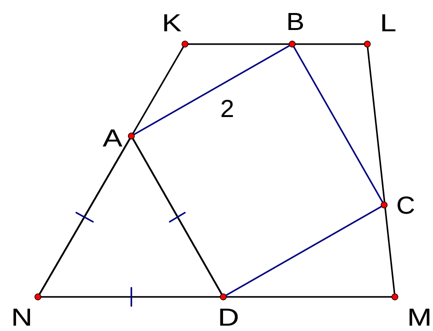

Figure 3

4. Two people, both starting at sunrise, walk toward each other from opposite ends of a hiking trail. They meet at noon. Both continue hiking, the faster one finishing the trail at 5:20 pm, the slower one finishing at 8:20 pm. What time was sunrise?

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5. *Figure 5* shows square *ABCD*, of side length 2, inscribed in cyclic quadrilateral *CRLE* so that ΔALD is equilateral. Compute the length of diagonal $\overline{LC}$.

LC = ____________________

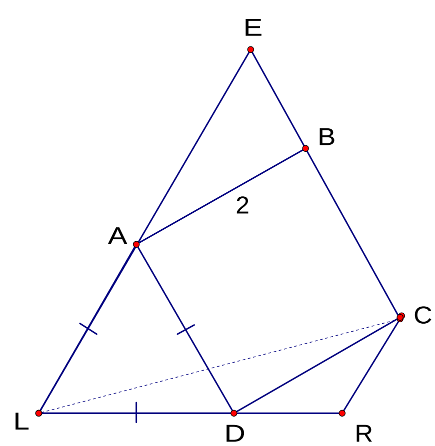

Figure 5

6. Alice, Beth, and Cindy, starting together, walk in the same direction around a circular track. It takes the girls $5/36$, $2/9$, and $35/99$ of an hour, respectively, to walk once around the track. How many laps will Alice have walked by the next time all three girls are together again at the starting point?

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Team ____________________