

Minnesota State High School Mathematics League Individual Event

2009-10 Event 1A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Express $\frac{\frac{2}{3} + \frac{1}{2}}{2} + \frac{1}{\frac{2}{3} + \frac{1}{2}}$ as the quotient of two relatively prime integers.

2. Alec is hosting a large party and needs to buy 100 bottles of Sierra Dew. Luckily, CostCorp sells them in bulk! He can buy 36 bottles for \$9.00, 15 bottles for \$4.50, or 4 bottles for \$1.60. What's the smallest amount Alec could pay, if he were to buy exactly 100 bottles?

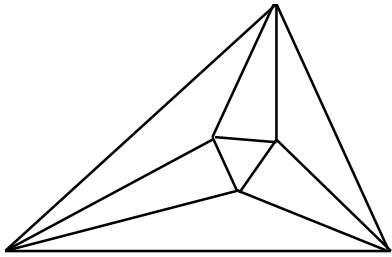
3. Find integers m and n , with $m < n$, for which the least common multiple of m and n is 105 and $50 < m + n < 70$.

$m =$ _____ $n =$ _____

4. In order to cope with the recession in auto sales, Acana Mist keeps adjusting the price for which she is selling her used car, which is worth \$10,000. Starting at this price prior to day 1, each day Acana either raises or lowers the price by 20% (but never asks less than the car is worth). Eventually, after d days of adjusting the price, she finds a buyer and makes a small profit – less than 5% of the car's worth. What is the smallest possible value for d ?

Name _____

Team _____



Minnesota State High School Mathematics League Individual Event

2009-10 Event 1B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. In *Figure 1*, $\triangle ABC$ is an isosceles triangle with $AB = AC$. Find $m\angle DAC + m\angle BCF + m\angle CBE$.

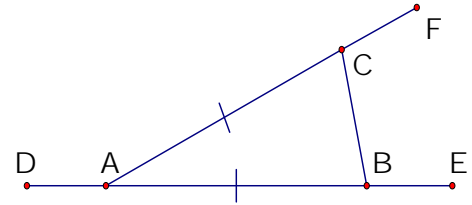


Figure 1

2. On hypotenuse \overline{PR} of right triangle PQR (*Figure 2*), X is the point for which $PX = PQ$. If $m\angle P = 40^\circ$, find $m\angle RQX$.

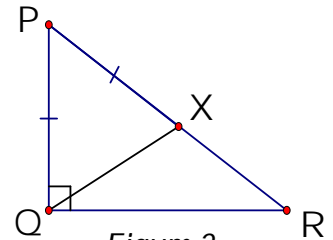


Figure 2

3. In the regular 9-sided polygon shown in *Figure 3*, find the measure of the acute angle formed by the extensions of the two darkened sides.

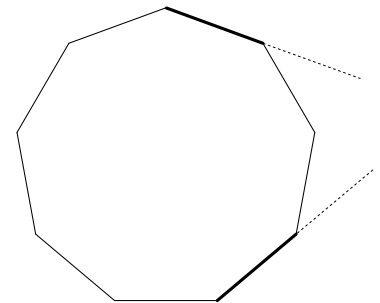


Figure 3

4. In *Figure 4*, $AB = BC = CD = DE = EF = FG = GA$. Find $m\angle A$.

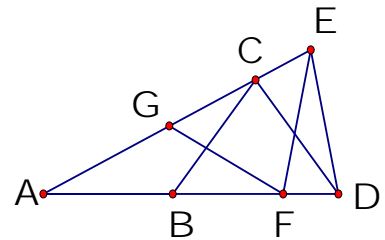
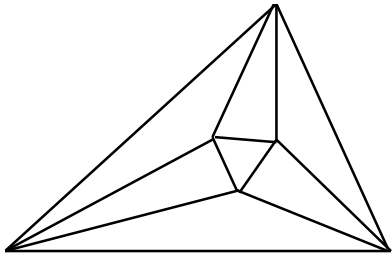


Figure 4

Name _____

Team _____



Minnesota State High School Mathematics League Individual Event

2009-10 Event 1C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. In *Figure 1*, $AB = 4$, $BC = 3$, and $AC = 5$. Find $\cot A$.

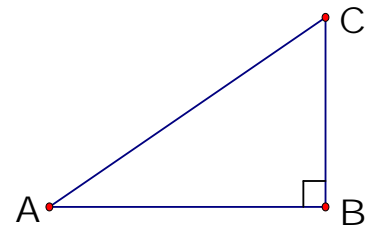


Figure 1

2. Again use *Figure 1*, but this time ignore the values from problem #1. Now, $\sin A = \frac{3}{5}$ and $AC = 7$. Find the length of \overline{BC} .

3. In *Figure 3*, $\sin \angle EDF = \frac{3}{5}$ and $\sin \angle DEF = \frac{4}{5}$. If $DF = 7$, find the length of \overline{EF} .

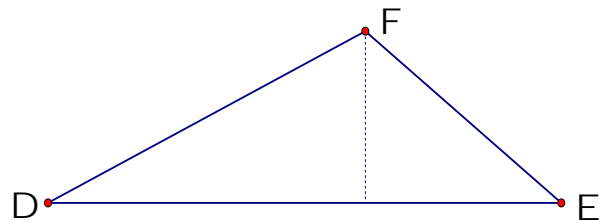
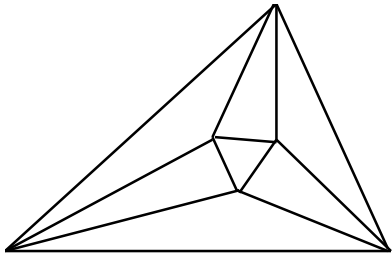


Figure 3

4. Again in *Figure 3*, $\sin \angle EDF = \frac{3}{5}$ and $\sin \angle DEF = \frac{4}{5}$, but DF does not necessarily equal 7. If $DE = 10$, find the length of the altitude dropped from F .

Name _____

Team _____



Minnesota State High School Mathematics League Individual Event

2009-10 Event 1D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. In the following synthetic division, what number belongs in the empty box?

$$\begin{array}{r|rrrr} -2 & 1 & 7 & \boxed{} & -9 \\ & & 1 & 5 & -6 \\ \hline & & & & \boxed{3} \end{array}$$

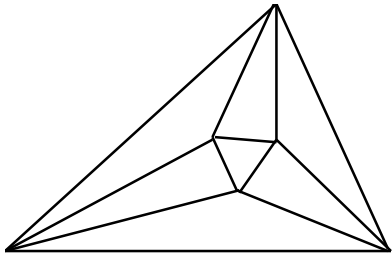
2. Find the remainder when $2x^3 - x + 6$ is divided by $x - 5$.

3. If $f(x) = x + 1$, $g(x) = x^2 + 2$, and $h(x) = x^3 + 3$, write an expression for $f(g(h(x)))$ as a standard-form polynomial in x .

4. For how many integers $c \leq 2009$ will the solutions of the equation $x^2 - 38x + c = 0$ be complex conjugates of the form $a \pm bi$, where a and b are positive integers?

Name _____

Team _____



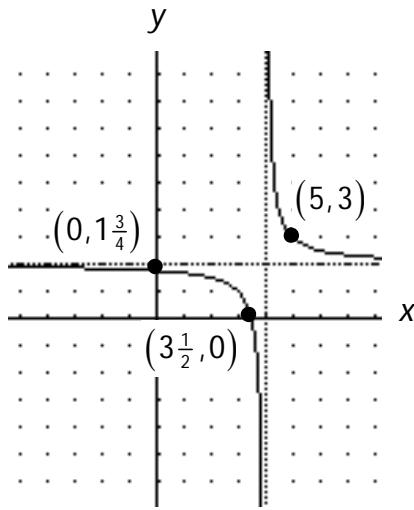
Minnesota State High School Mathematics League

Team Event

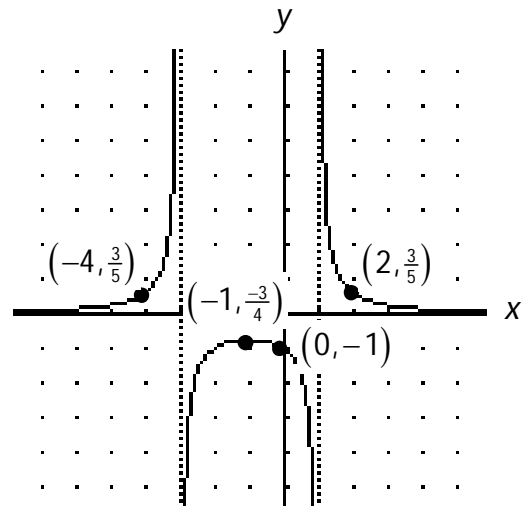
2009-10 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1.



$$f(x) = \frac{a}{x-c} + d$$



$$g(x) = \frac{p}{(x-q)(x-r)}$$

The graphs of $f(x)$ and $g(x)$, two rational functions, are shown. The function $(f+g)(x)$ has exactly one rational root. Find it.

2. The greatest common divisor of a , b , and c is 6. The greatest common divisor of ab , ac , and bc is 360. What is the smallest possible value for abc ?

3. For a circle of radius r , the ratio of the side length of a regular circumscribed polygon of n sides to the side length of a regular inscribed polygon of n sides may be written in the form $T(f(n))$, where T is one of the six standard trigonometric functions and f is a function involving π and n . Do so.

Team _____

2009-10 Meet 1 Team Event
(page 2)

4. If $f(x) = x + 1$, $g(x) = x^2 + 2$, $h(x) = x^3 + 3$, and $k(x) = x - c$, find the smallest value of c that causes $k\left(f\left(g\left(h(x)\right)\right)\right)$ to have a real root.
-

5. In parallelogram $PQRS$, angle PSR is acute, and point X is located on side \overline{RS} so that $XR = RQ$, $XQ = QP$, and $XP = PS$. Find $m\angle PSX$.
-

6. Express $\frac{5}{0.\overline{2} + 0.\overline{4} + 0.\overline{7}}$ as the quotient of two relatively prime integers.
-

Team _____