Minnesota State High School Mathematics League

Individual Event

2008-09 Event 2A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Given that $(y+7)+(2y+x^2)=(3+x^2)+3$, find the value of y.

2. The Sir Charge car rental company charges \$45 per day to rent a car, plus \$6.00 per gallon of gas used. The car can drive 26 miles per gallon. If Teddy rents this car from Sir Charge for 5 days and drives it a total of m miles, write a linear expression in terms of m which describes Teddy's total cost.

3. Don's calculator has some warped circuits and does not input integers correctly. When Don enters an odd integer n, the calculator interprets it as $n + 3$. When Don enters an even integer m, the calculator interprets it as $m/2$. If Don entered the numbers 15, 24, 33, and x, and the calculator claimed the sum of those numbers was 101, find x.

4. Ole, who occasionally lies, wrote down an integer N, and then gave his friend Lena the following inequalities:

   A. $N-13<50-N$
   B. $N+8<100-2N$
   C. $3N+60<5N+3$
   D. $5N-20<N+99$

   As it turned out, exactly two of those statements were lies. Find the value of N.

Name

Team

Minnesota State High School Mathematics League

Individual Event

2008-09 Event 2B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. The lengths of the sides of a scalene triangle, listed in size order, are 5, x, and 15. How many possible values for x are there, given that x is an integer?

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2. Point P is chosen along leg $\overline{BC}$ of right triangle ABC so that $BP = PA$ *(Figure 2).* If $BC = 10$ and $AC = 4$, find BP.

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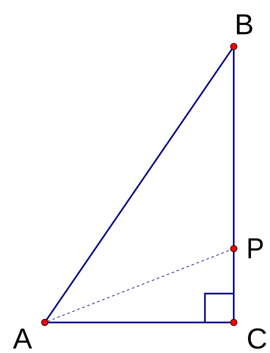

Figure 2

3. In an isosceles triangle with a 30° vertex angle, the perpendicular bisector of one leg divides the other leg into the ratio $k : 2$. If $k \geq 2$, find k.

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4. In triangle ABC, $m\angle ABC = m\angle ACB = 72°$. Find the ratio $AB : BC$, and express it as a decimal accurate to three decimal places.

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Name __________________________ Team __________________________

Minnesota State High School Mathematics League

Individual Event

2008-09 Event 2C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

Questions 1 and 2 refer to Figure 1 in which $\tan\alpha = \frac{5}{12}$, $\tan\beta = 4\sqrt{5}$.

1. Find $\sin 2\alpha$.

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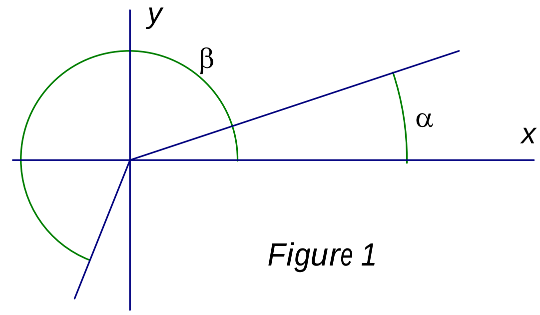

Figure 1

2. Find $\cos\frac{\beta}{2}$.

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3. In Figure 3, $\theta_1 + \theta_2 = 45°$. Find BC.

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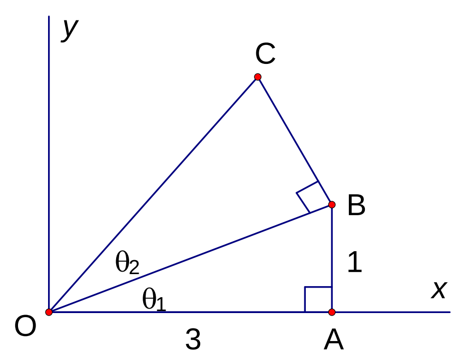

Figure 3

4. *ABCD* is a kite-shaped quadrilateral *(Figure 4)* with *BC* = *CD* and *AB* = *AD* = *x*. $m\angle BAD = 45°$; $m\angle BCD = 135°$. From *D*, drop a line perpendicular to $\overline{AB}$, meeting $\overline{AC}$ at *E*, and $\overline{AB}$ at *F*. Find *EF* in terms of *x*.

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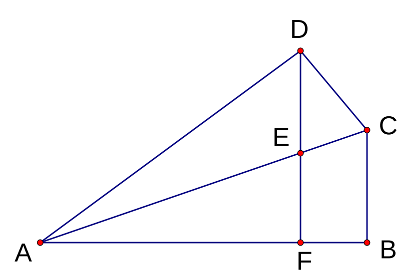

Figure 4

Name ____________________________ Team ____________________________

Minnesota State High School Mathematics League

Individual Event

2008-09 Event 2D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. Write the equation of the line which passes through the origin and is parallel to $2x-3y=7$.

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2. Find the intersection point of the lines $y=\frac{7}{5}x-10$ and $y=\frac{3}{4}x$.

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3. Find all ordered pairs (x,y) that satisfy

$$\begin{cases} x^2 + y^2 = 5 \\ x^2+(y+4)^2 = 6^2 \end{cases}$$

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4. Two lines intersect in the *xy*-plane. The first line has *x*-intercept 2*p* and *y*-intercept 2*p*, while the second line has *x*-intercept *p* and *y*-intercept 3*p*. If their intersection point is concurrent with the line *x* = 3, find *p*.

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Name ______________________________ Team ______________________________

Minnesota State High School Mathematics League

Team Event

2008-09 Meet 2

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. If a, b, and c are distinct in the system $\begin{cases} a^3 + 3a = -14 \\ b^3 + 3b = -14 \\ c^3 + 3c = -14 \end{cases}$, find the value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

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2. The altitude and the median from vertex A of triangle ABC are 4 and 5 units long, respectively. The altitude bisects the angle determined by side $\overline{AB}$ and the median. Find AC.

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3. Referring to problem #3 from Event A, Don bought a new calculator to replace his old one. Sadly, this new calculator also inputs integers incorrectly. It interprets the odd integer n as $4n$, and the even integer m as $m + 5$. When Don used the new calculator to multiply the positive integers a and b, the result was 4444. Find the smallest possible value that the <u>calculator</u> would return for the sum $a + b$.

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4. Susan, who had no calculator available, was asked to solve $\sqrt{6}\cos x + \sqrt{2}\sin x = 2$. She squared both sides and simplified to get the equation $4\cos^2 x + b\sin 2x = 2$. In exact form, what is b ?

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5. Points A and B are on the same side of line m and are 5 and 7 units away from m, respectively. A and B are 17 units apart. For all points P on line m, what is the smallest possible value of $AP + BP$?

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6. In a triangle, the lengths of the three medians are 9, 12, and 15. Find the length of the side to which the longest median is drawn.

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Team __________________________