

Minnesota State High School Mathematics League

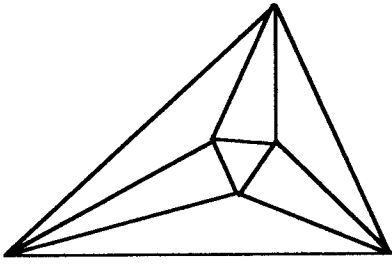
Individual Event

2008-09 Event 1A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. Express $\frac{1}{2} + \frac{1}{4} + \frac{1}{12}$ as the quotient of two relatively prime numbers.
- _____ 2. Express 12.5% of $\frac{.0032}{.0018 + .0003}$ as the quotient of two relatively prime numbers.
- _____ 3. [Here is a slight modification of a problem credited to the well known mathematician, Paul Halmos] A watermelon weighs 500 pounds, 99% of its weight being due to the water it contains. After it sat in a drying room for a while, it lost 250 pounds of water. What percent of its weight was then water?
4. Three positive integers L , M , and N satisfying $L < M < N$, have a greatest common divisor of 12 and a least common multiple of 180. Find all possible triples (L, M, N) .
- _____

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2008-09 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. The isosceles $\triangle ABC$ in Figure 1 has vertex $\angle C = 40^\circ$. A point D is chosen on BC so that $AD = AB$. What is the measure in degrees of $\angle DAC$?
- _____ 2. In the right $\triangle ABC$, the bisectors of the acute angles B and C meet at D to form an isosceles $\triangle BCD$. What is the measure in degrees of $\angle BDC$?
- _____ 3. The vertices of a regular pentagon, labeled in a counterclockwise direction, are $ABCDE$. What is the angle measure of $\angle DAE$?
- _____ 4. In quadrilateral $ABCD$ (Figure 4), $\angle ABC = 42^\circ$. Furthermore, if AB is extended to E so that $AB = BE$, then $\angle ACE = 90^\circ$. What is the measure of $\angle AEC$?

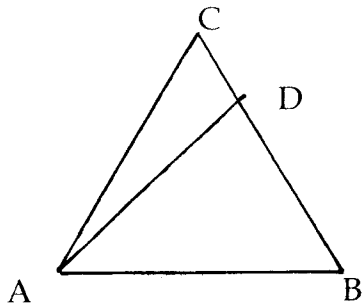


Figure 1

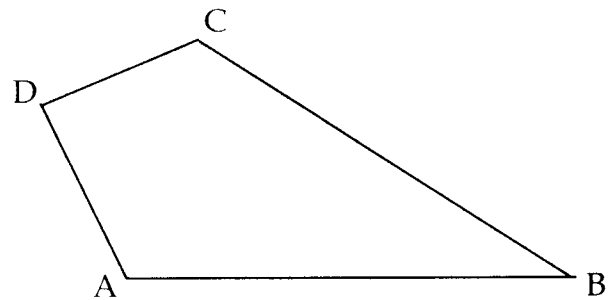
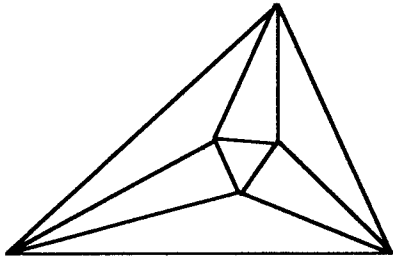


Figure 4

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2008-09 Event 1C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. For the second quadrant angle pictured in Figure 1, find $\sin \alpha + \tan \alpha$.

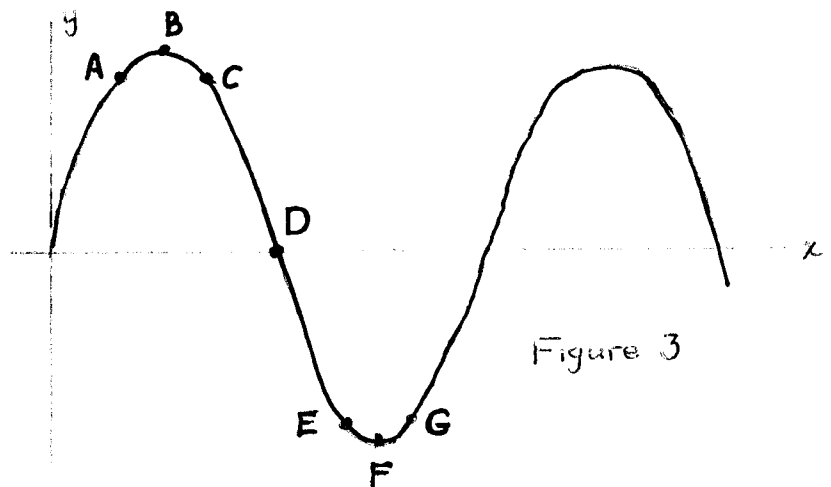
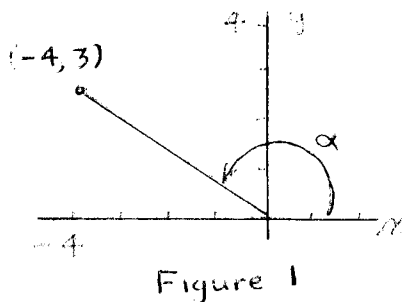
_____ 2. Express as a multiple of π the radian measure of an angle in the third quadrant that has a sine of $-\frac{1}{2}$.

3. Figure 3 shows the graph of $y = 2 \sin \frac{4}{3} x$. No scale markings are shown on the axes, but certain points on the graph have been labeled. Write the label of the point on the graph having an x -coordinate of

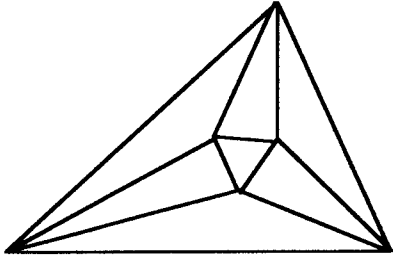
(a) π _____

(b) $\frac{\pi}{2}$ _____

_____ 4. Round to the nearest multiple of 50 the number of x intercepts on the graph of $y = \sin \frac{1}{x}$ when $0.0001 < x < 0.001$. That is, to the nearest 50, how many times will the graph of $y = \sin \frac{1}{x}$ cross the x -axis between 0.0001 and 0.001?



Name _____ Team _____



Minnesota State High School Mathematics League Individual Event

2008-09 Event 1D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS ON THIS EVENT

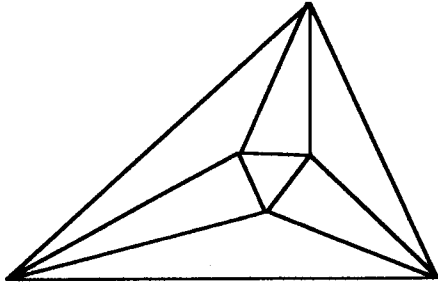
_____ 1. Find all the solutions to $x^2 + 4x + 4 = 9$.

_____ 2. Find all the solutions to $(2x - 3)(x + 1) = 42$.

3. Write the equation of the parabola passing through $(3,7)$, $(1,4)$ and $(5,4)$. Write your answer in the form $y = ax^2 + bx + c$ OR $x = ay^2 + by + c$, whichever form fits the situation.

_____ 4. Find the smallest root of $6x^3 - 13x^2 - 19x + 12 = 0$.

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2008-09 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

- _____ 1. Isosceles $\triangle ABC$ (Figure 1) has base angles $\angle A = \angle B = 70^\circ$. AE makes an angle of θ with AB , and θ varies as E moves up and down BC . DE is parallel to AB , and of course it too moves up or down with E . The extensions of DE to DF and AE to AG form angles $\alpha = \angle FEG$ and $\beta = \angle GEC$. What will be the measure of θ when $\alpha = \beta$? [AHSME, 1968, Number 18]
- _____ 2. Having purchased 200 shares of a stock at one price, and another 200 shares at a higher price, Mr. Gotbucks later sold all 400 shares for \$30 each. He thereby gained 20% on the first 200 shares, but lost 20% on the other 200 shares. How much did he gain or lose at the time of the sale?
- _____ 3. How many ordered pairs (a,b) of positive integers exist such that $\frac{1}{a} + \frac{5}{b} = \frac{1}{2}$?
- _____ 4. Find the measure in degrees of the sum of angles A, B, C, D, E and F in Figure 4. [AHSME, 1972, Number 21]
- _____ 5. Consider the set of composite positive integers between 47 and the next largest prime. Let L be the least common multiple of this set, and let S be the largest integer such that S^2 is a factor of L . What is the value of $\frac{L}{S^2}$?
- _____ 6. In $\triangle ADE$, $\angle ADE = 140^\circ$, points B and C lie on sides AD and AE respectively, and point $A, B, C, D,$ and $E,$ are distinct. If $AB = BC = CD = DE$, what is the measure of $\angle EAD$? [AHSME, 1978, Number 12]

Team _____

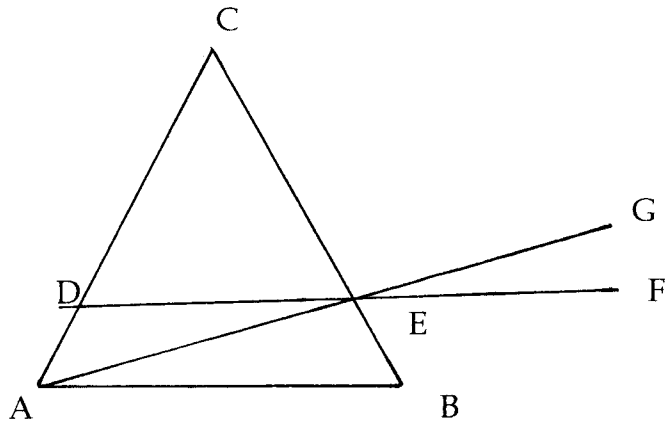


Figure 1

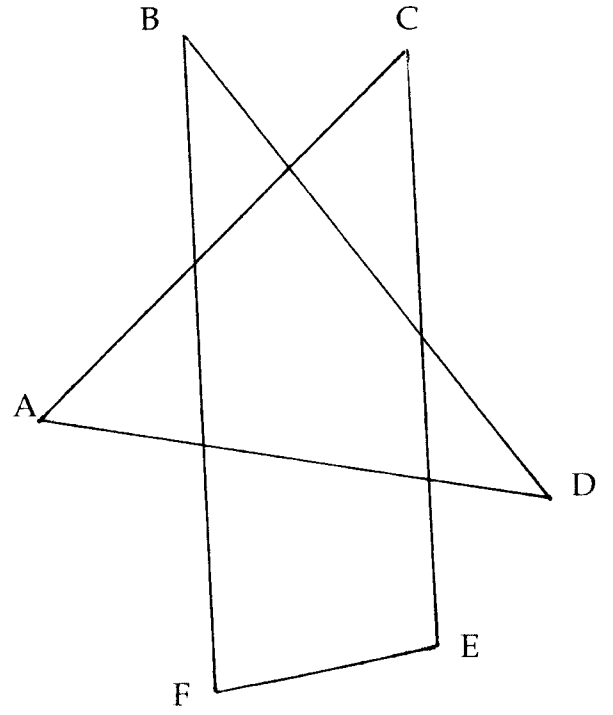


Figure 4