

# Minnesota State High School Mathematics League Individual Event

## 2008-09 Event 5A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have **20 minutes** for this event.

**NO CALCULATORS are allowed on this event.**

1. The six-digit number  $\underline{2} \underline{1} \underline{7} \underline{X} \underline{8} \underline{5}$ , when divided by 9, leaves a remainder of 2. What is the value of the obscured digit,  $X$ ?

$X =$  \_\_\_\_\_

2. A single digit is placed in each empty square in the grid (Figure 2) so that each row and each column contain exactly one of each of the digits 1, 2, 3, 4, and 5. What digit must be placed in the square at the bottom right corner?

	5	4		
1	3			
		5	3	
2		3	1	
				?

Figure 2

3. In the addition problem shown in Figure 3, each letter stands for a distinct non-zero digit. If the problem is mathematically correct, what is the largest possible value for the three-digit number **ONE**?

$$\begin{array}{r}
 \text{ONE} \\
 + \text{ONE} \\
 \hline
 \text{TWO}
 \end{array}$$

$\text{ONE} =$  \_\_\_\_\_

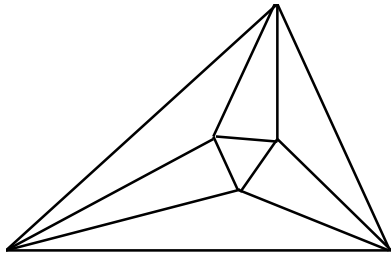
Figure 3

4.  $N$  is a five-digit positive integer in which the first and last digits are unequal and nonzero.  $M$  is the five-digit integer obtained by writing the digits of  $N$  in reverse order. If  $M > N$ ,  $D = M - N$ , and the last three digits of  $D$  are  $\underline{9} \underline{7} \underline{6}$ , find  $D$ .

$D =$  \_\_\_\_\_

Name \_\_\_\_\_

Team \_\_\_\_\_



# Minnesota State High School Mathematics League Individual Event

## 2008-09 Event 5B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. The sides of square  $ABCD$  have length 3. Points  $R$ ,  $S$ ,  $T$ , and  $U$  partition the sides into lengths in the ratio 2:1, as shown in *Figure 1*. What is the area of  $\triangle ART$ ?

$\text{Area}(\triangle ART) =$  \_\_\_\_\_

2. Again using *Figure 1* (as described in problem 1), find the area of  $RSTU$ .

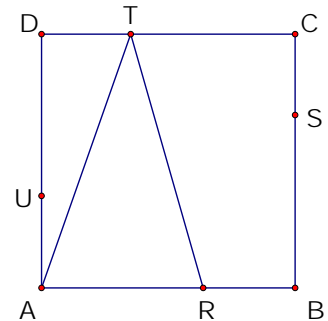
$\text{Area}(RSTU) =$  \_\_\_\_\_

3. In *Figure 3*, each edge of square  $JKLM$  is partitioned into thirds. Four line segments are then drawn, connecting the partition points and forming an inner square  $WXYZ$ . Express, as a fraction in lowest terms, the portion of the area of  $JKLM$  covered by  $WXYZ$ .

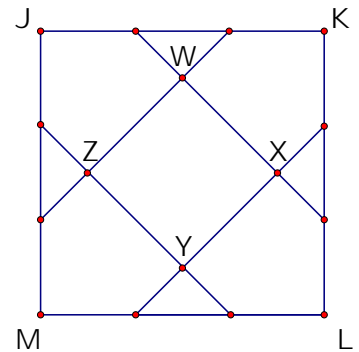
\_\_\_\_\_

4. A cube of side length  $s$  is inscribed in a sphere, which is itself inscribed in a cone with slant height equal to the diameter of its base. Find the volume of the cone, expressed in terms of  $s$ .

\_\_\_\_\_



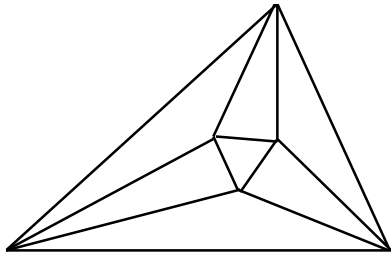
*Figure 1*



*Figure 3*

Name \_\_\_\_\_

Team \_\_\_\_\_



# Minnesota State High School Mathematics League Individual Event

## 2008-09 Event 5C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. How many distinct triangles exist within the MN State High School Math League logo at the top of this page?

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2. Find the number of positive-integer factors of 2600.

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3. At a carnival game, a black velvet pouch contains either a gold or silver coin, with equal probability of each. You win if you can correctly guess the color of the coin. While you are contemplating your choice, the game operator says, "Let me help you." He drops in two gold coins, shakes the pouch, then reaches in and draws out... two gold coins. What is the probability the remaining coin is gold?

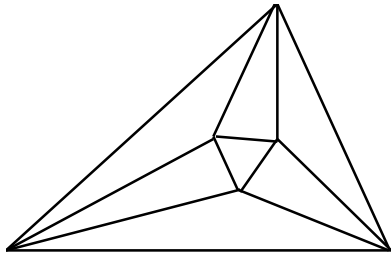
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4. Fourteen people are in line at a cashier's office. Seven owe \$1.00, and the other seven are expecting a refund of \$1.00. Unfortunately, the cashier has an empty cash box, and cannot give refunds unless he receives money first. Given a random arrangement of people in line, what is the probability that the cashier can serve them all, in order, without ever dropping below \$0?

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Name \_\_\_\_\_

Team \_\_\_\_\_



# Minnesota State High School Mathematics League Individual Event

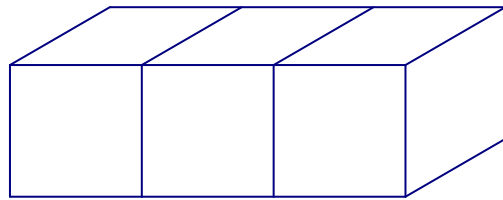
## 2008-09 Event 5D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. If  $\frac{3}{4}$  of 8 apples are worth exactly as much as  $\frac{7}{8}$  of 16 oranges, how many oranges can I get for the same amount of money that buys 75 apples?

\_\_\_\_\_

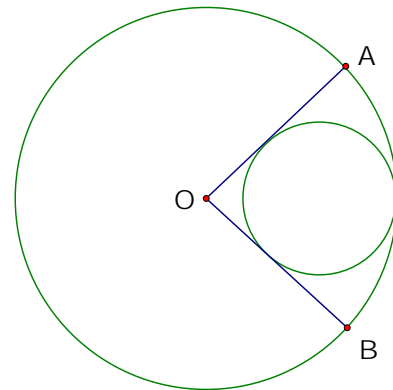
2. The six faces of a cube are labeled with the six numbers 1, 2, 4, 8, 16, and 32, in some order. Three identical such cubes are placed on a table touching each other, as shown in *Figure 2*. Find the largest possible sum of the 11 visible faces.



*Figure 2*

\_\_\_\_\_

3. Points  $A$  and  $B$  are on a circle centered at  $O$ . A second circle is internally tangent to the first, and tangent to both  $\overline{OA}$  and  $\overline{OB}$  (*Figure 3*). If the ratio between the radii of the two circles is  $\frac{3}{7}$ , compute  $\cos \angle AOB$ .



*Figure 3*

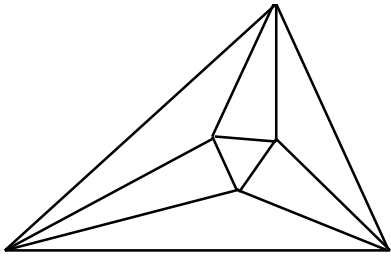
$\cos \angle AOB =$  \_\_\_\_\_

4. Let  $k = 2^{2009} + 2009^2$ . Compute the units digit (ones' place) of  $k^{2009} + 2009^k$ .

\_\_\_\_\_

Name \_\_\_\_\_

Team \_\_\_\_\_



# Minnesota State High School Mathematics League Team Event

## 2008-09 Meet 5

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1.  $N$  is a five-digit positive integer in which the first and last digits are unequal and nonzero.  $M$  is the five-digit integer obtained by writing the digits of  $N$  in reverse order. If  $M > N$ ,  $D = M - N$ , and the last three digits of  $D$  are 029, find  $D$ .

$D =$  \_\_\_\_\_

2. In  $\triangle ABC$  (Figure 2),  $\overline{AB}$  and the perpendicular bisector of  $\overline{BC}$  meet at  $D$ , making  $AD = 9$  and  $DB = 7$ . It also happens that  $\overline{CD}$  bisects  $\angle ACB$ . Find the area of  $\triangle ACD$ .

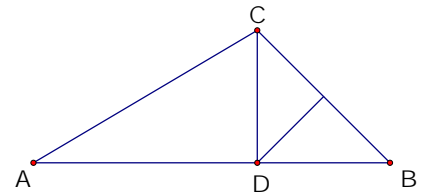


Figure 2

$\text{Area}(\triangle ACD) =$  \_\_\_\_\_

3. How many distinct quadrilaterals exist within the MN State High School Math League logo at the top of this page?

\_\_\_\_\_

4. Find the surface area of a cube whose volume is twice that of a cube with surface area 1. Express your answer in simplest radical form.

\_\_\_\_\_

5. A rearrangement of the ten numbers 1, 1, 3, 3, 5, 5, 7, 7, 9, 9 is called "nerdy" if the four-digit sequence 1, 3, 3, 7 occurs left-to-right *somewhere* in the sequence (but not necessarily consecutively). Compute the number of nerdy rearrangements.

\_\_\_\_\_

6. Right triangle  $PQR$  has perimeter 60. The altitude dropped from  $P$  to the hypotenuse has length 12. Find the area of  $\triangle PQR$ .

$\text{Area}(\triangle PQR) =$  \_\_\_\_\_

Team \_\_\_\_\_