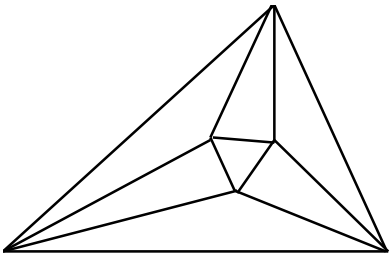


Minnesota State High School Mathematics League

Individual Event

2008-09 Event 3A

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. Solve for x: $\begin{cases} x+y=11 \\ 2x+y=50 \end{cases}$

x = _____

2. *Definition: A <u>lattice point</u> is a point in the xy-plane with integer coordinates.*

   Find the only lattice point in the 1st quadrant (and therefore, not lying on an axis) which is a solution of the system:

$$\begin{cases} y < \frac{1}{2}x - 1 \\ y < \frac{-2}{3}x + 5 \end{cases}$$

(x, y) = _____

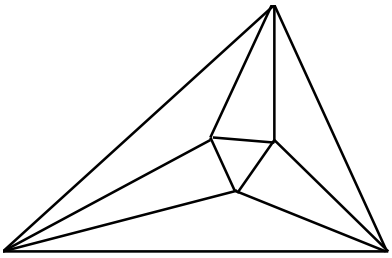
3. If $\begin{vmatrix} n & 2n & 3n \\ 1 & 0 & 2 \\ 0 & 3 & 0 \end{vmatrix} = 6$, find n.

n = _____

4. ShaKiela and Wei-Chi share a birthday today. In three years, ShaKiela will be four times as old as Wei-Chi was when ShaKiela was two years older than Wei-Chi is today. If Wei-Chi is a teenager, find ShaKiela's age.

_____

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2008-09 Event 3B

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Rhombus *ABCD (Figure 1)* has sides of length 13. One of its diagonals has length 10. Find the area of *ABCD*.

_____

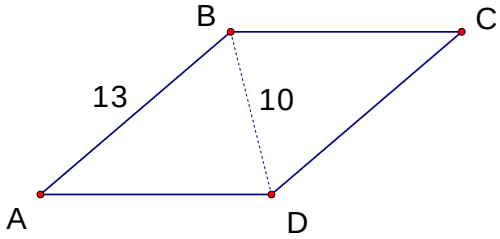


Figure 1

2. Two distinct diagonals are drawn inside a regular 30-gon. They intersect at the center of the polygon, *P*, and form both acute and obtute angles at *P*. What is the largest possible degree measure of the obtuse angle?

_____

3. A trapezoid has bases of lengths 4 and 8, and sides of lengths 3 and 5. Find the area of the trapezoid.

_____

4. Concave hexagon *ABCDEF* is formed by attaching rhombi *ABCD* and *ADEF* along edge $\overline{AD}$, as shown in *Figure 4*. Given that *A* lies on $\overline{BE}$, *BD* = 5, and *DF* = 6, find the area of *ABCDEF*.

_____

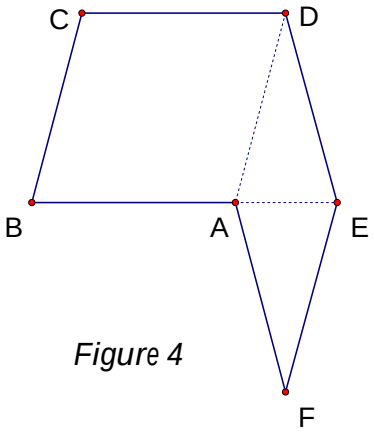
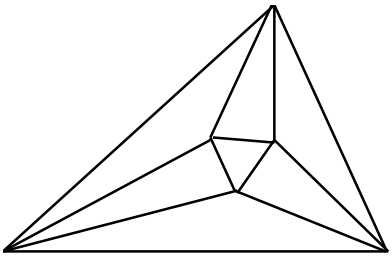


Figure 4

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2008-09 Event 3C

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. Find the solution of the equation $\cos x - \sin x = 0$ where $\pi \le x < \frac{3\pi}{2}$.

***x* =** _____

2. Find all solutions to the equation $\sec^2\theta - 3\sec\theta - 2 = 0$ on the interval $0 \le \theta < 2\pi$.

***θ*=** _____

3. In parallelogram *ABCD*, $\overline{AC}$ and $\overline{BD}$ are diagonals. If *BC* = 7, *AB* = 8, and $m\angle C = 60°$, find the value of $AC^2 - BD^2$.

AC^2 - BD^2 = _____

4. In *Figure 4*, $m\angle ABC = 120°$ and *AB* = 1. Lying on side $\overline{BC}$ are the points B_1, B_2, B_3, ..., B_n such that the distance from *A* to B_k is $\sqrt{k}$. If the distance from B_k to B_{3k} is 2, find *k*.

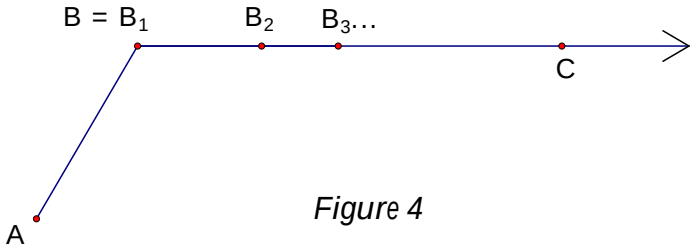
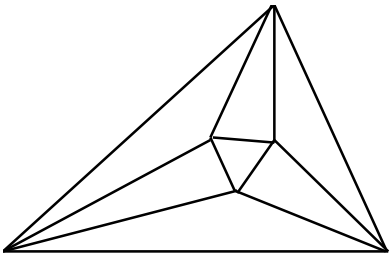


Figure 4

***k* =** _____

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2008-09 Event 3D

Question #1 is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

NO CALCULATORS are allowed on this event.

1. Given that $2\log_2 x = 6$, find x.

x = _____

2. If $\sqrt{x \cdot \sqrt[5]{x}} = x^h$, find h.

h = _____

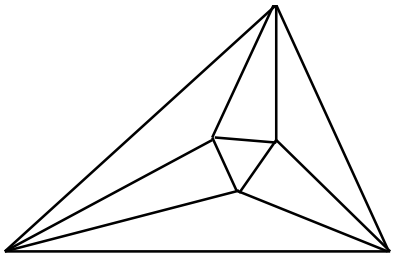
3. Suppose a = log 5, and b = log 9, where the logarithms are base 10. Find log 12 in terms of a and b.

log 12 = _____

4. Find the sum of all positive integers N for which $\left[\sqrt{9+\sqrt{N}} - \sqrt{9-\sqrt{N}}\right]^2$ is an integer.

_____

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2008-09 Meet 3

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. Convex pentagon *ABCDE* is inscribed in a circle of radius 1. *AB* = *BC*, *CD* = *DE* = *EA*, and *AC* = 2. Find *BD*.

***BD* =** _____

2. Given that $\log_{12} 3 = x$ and $\log_{12} 75 = y$, find $\log_{12} \frac{40}{9}$ in terms of *x* and *y*.

_____

3. Lines ℓ_1, ℓ_2, and ℓ_3 create $\triangle ABC$ *(Figure 3)*. The incircle has center *I*. A circle tangent to all three lines, but on the other side of ℓ_1 from *C*, has center *K* and radius 12. Given that *AK* = 13 and *BK* = 15, find *KI*.

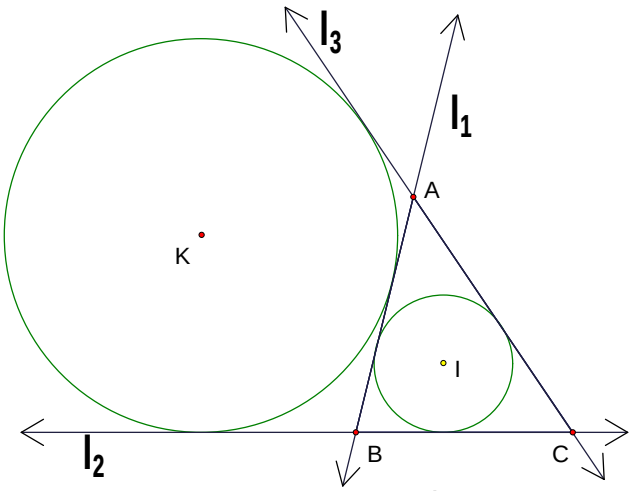


Figure 3

***KI* =** _____

4. Four positive integers sum to 125. If you increase the first of these numbers by 4, decrease the second by 4, multiply the third by 4, and divide the fourth by 4, you produce four equal numbers. Find the four integers, listing them in the order presented in the problem.

{ , , , } _____

5. In parallelogram *ABCD (Figure 5)*, the bisector of $\angle ABC$ intersects $\overline{AD}$ at point *P*. If *PD* = 5, *BP* = 6, and *CP* = 6, find *AB*.

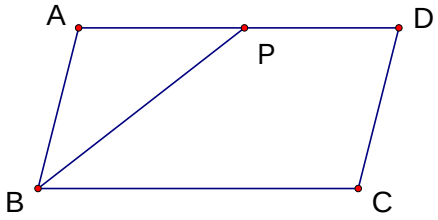


Figure 5

***AB* =** _____

6. If $\left[\log_2(4)-1\right]+\left[\log_2(6)-1\right]+\left[\log_2(8)-1\right]+\ldots+\left[\log_2(2008)-1\right]=\log_2(k!)$, find *k*.

***k* =** _____

Team _____