Minnesota State High School Mathematics League

Individual Event

2007-08 Event 2A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. One day, a precocious young elementary student got bored with addition and multiplication, so she invented a new operation, written ‡. She defined it as:

$$a \ddagger b = \frac{ab}{a+b}$$

Find 27 ‡ 6, writing your answer as the quotient of two relatively prime integers.

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2. Federal income tax for most people is determined by using two-column tables. These tax tables are created using linear equations that relate the tax a person owes (T) to their income (I). Using the tax table shown at the right, find the linear equation for T, in terms of I.

Income(I)	**Tax(T)**
24,000	3226
24,100	3241
24,200	3256
24,300	3271

T = __________________________

3. You are doing a "mental math" trick. You ask a friend to choose a number N without revealing it to you. Then you tell your friend to subtract 5 from it, triple the result, add 6, multiply by 4, subtract the original number, and then take 12.5% of what remains. Then you ask your friend for the final result – call it R.
Find an equation in the form N = pR + q, where p and q are quotients of relatively prime integers, that allows you to determine N knowing only the value of R.

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4. The integers *m* and *n* each have exactly two prime factors, not necessarily distinct. The difference between the LCM and the GCD of *m* and *n* is 102. Find *m* and *n*.

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Name ____________________________ Team ____________________________

Minnesota State High School Mathematics League

Individual Event

2007-08 Event 2B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event. **NO CALCULATORS are allowed on this event.**

1. In right triangle ABC (Figure 1), C is the right angle, $\overline{DE} \parallel \overline{BC}$, $m\angle A = 30°$, $AE = 2$, and $EC = 7$. Find the length BC.

2. Consider the triangle with side lengths $3x + 91$, $4x + 19$, and $5x + 1$. What is the <u>largest</u> value of x for which this triangle will be isosceles?

3. In Figure 3, $QR = 3$, $RP = 4$, $PQ = 5$, and $PM = MN = NQ = x$.
Find x, writing your answer in the form $a - b\sqrt{c}$, where a, b, and c are integers greater than 1.

4. In triangle XYZ, the median from Y meets $\overline{XZ}$ at G, and the median from Z meets $\overline{XY}$ at H. If these medians are perpendicular, and $XG : XH = 3 : 4$, find the ratio $GH : HY$.

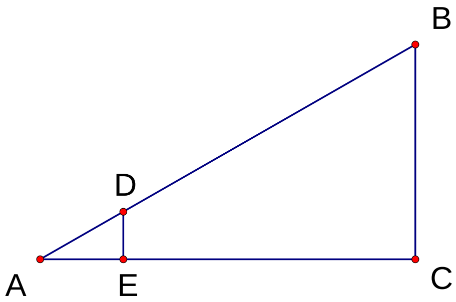

Figure 1

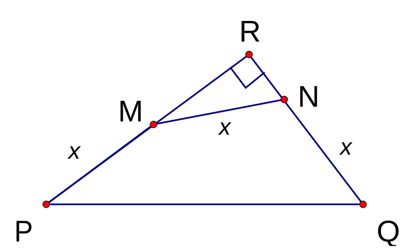

Figure 3

Name ______________________ Team ______________________

Minnesota State High School Mathematics League

Individual Event

2007-08 Event 2C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event. **NO CALCULATORS are allowed on this event.**

In this event, all angles are measured in degrees.
The answer to each problem is the smallest positive x satisfying the given equation.

(Several problems make use of the fact that in a right triangle ABC, the sine of one acute angle and the cosine of the other acute angle are related.)

1. $\sin(2x+8)=\cos(5x-9)$

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2. $\sin x = -\sqrt{\dfrac{1+\sin 38^\circ}{2}}$

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3. $\sec 5x = \tan 3x + \tan 5x$

   (Hint: As your first step, try turning each term into a fraction.)

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4. $(\cos 8x)(\cos 4x)(\cos 2x)(\cos x)=\frac{1}{16}$

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Name ______________________________ Team ______________________________

Minnesota State High School Mathematics League

Individual Event

2007-08 Event 2D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. If the graphs of $2y+x+3=0$ and $3y+ax+2=0$ are to meet at right angles, find the value of *a*.

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2. If a straight line intersects the x-axis at *r* and intersects the y-axis at $1/r$, at what point (x,y) does it intersect the line $y=x$?

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3. In the coordinate plane, a circle which passes through $(-2,-3)$ and $(2,5)$ <u>cannot</u> also pass through the point $(2007,y)$. Find the value of *y*.

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4. There are two circles that pass through $(1,9)$ and $(8,8)$ and are tangent to the x-axis. Find the lengths of their radii.

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Name ______________________________ Team ______________________________

Minnesota State High School Mathematics League

Team Event

2007-08 Meet 2

Each question is worth 4 points. Team members may cooperate in any way, but at the end of 20 minutes, submit only one set of answers. Place your answer to each question on the line provided.

1. Find an equation for a line parallel to, and 4 units away from, the line $y = \frac{3}{4}x + 6$.

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2. Consider all triangles whose side lengths are Pythagorean triples of the form (a, b, c), where $a < b < c$, a is an odd positive integer, and b and c are consecutive positive integers. Compute all possible values of the perimeter of such triangles, when expressed in base a.

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3. List <u>all</u> degree measures α, with $0° < \alpha < 180°$, such that the system of equations

$$\begin{cases} x \cdot \cos 11° - y \cdot \sin 149° = \sin\alpha \\ x \cdot \cos 101° + y \cdot \cos 31° = \cos\alpha \end{cases}$$

<u>requires</u> that x be equal to 1.

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4. Find all points common to the graphs of $(x - y + 2)(3x + y - 4) = 0$ and $(x + y - 2)(x - 5y + 7) = 0$.

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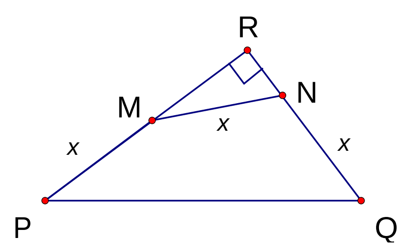
5. Problem #3 from Event B involved the diagram shown at right. If $QR = a$, $RP = b$, and $PQ = c$, then x can be expressed in the form $f(a,b) - \sqrt{g(a,b)}$, where f and g are first- and second-degree polynomials, respectively, both involving a and b. Find this expression for x.



x = __________________________

6. Sarah can deliver 60 papers in an hour and a half, while she and her friend Ryan, working simultaneously, can deliver 91 papers in 63 minutes. How long would it take Ryan to deliver 35 papers by himself?

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Team ____________________________