

Minnesota State High School Mathematics League

Individual Event

2007-08 Event 5A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 20 minutes for this event.

_____ 1. In a weight training program, a woman begins with a weight of w , and then, increasing in equal increments, she works her way up to x , then y , then z , and finally to a top weight of t . If $w + x + y + z + t = 45$, what is y ?

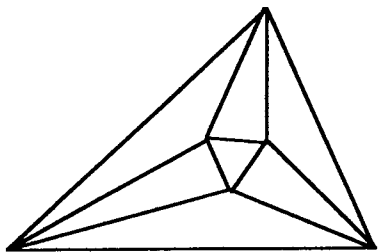
_____ 2. Four men were being questioned by the police about a robbery.
"Jack did it," said Alan
"George did it," said Jack.
"It wasn't me," said Sid.
"Jack is a liar if he said I did it," said George.

Only one had spoken the truth. Who committed the robbery?

_____ 3. The letters A , B , and C represent distinct digits. A is prime, $A - B = 4$, and the seven digit number $AAABBBBC$ is prime. What is that seven digit prime?

_____ 4. Rectangle $RSTU$ has sides of $RS = 2$, $ST = 1$. Five points placed interior to or on the boundary of this rectangle determine ten line segments. If the five points are placed so that the smallest of these ten segments is as large as possible, what will be the length of the shortest segment? Express your answer using decimal notation, rounded accurate to three places to the right of the decimal.

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2007-08 Event 5B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. What is the area of a $\triangle ABC$ that has sides of $AB = 4$, $BC = AC = 6$?
- _____ 2. The centers of a circle of radius 1 and an inscribed square coincide. What is the area of the region contained inside the circle, outside of the square?
- _____ 3. In the right $\triangle ABC$ (Figure 3), I is the center of the inscribed circle (the incenter). A line through I parallel to the hypotenuse intersects BC at D and AC at E , making $DI = 3$, $IE = 4$. Find the area of the circle.
- _____ 4. Draw an arbitrary $\triangle ABC$. As in Figure 4, extend each of AC , BA , CB to twice their length, forming $AZ = 2AC$, $BX = 2BA$, $CY = 2CB$. Express the area of $\triangle XYZ$ in terms of $a = \text{Area}(\triangle ABC)$. Hint: From C , drop altitude CV of $\triangle ABC$; then extend AB to W so that YW is an altitude of $\triangle XBY$

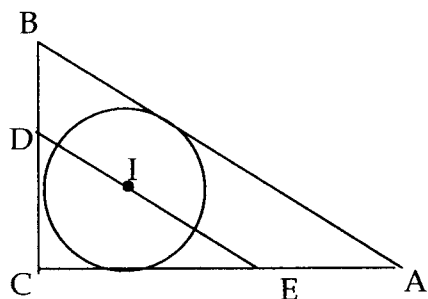


Figure 3

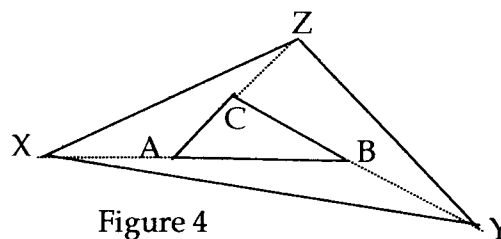
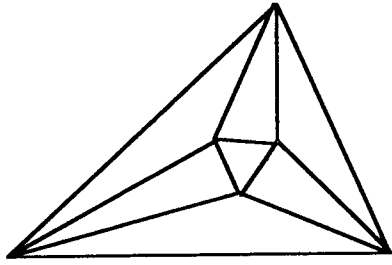


Figure 4

Name _____ Team _____



Minnesota State High School Mathematics League

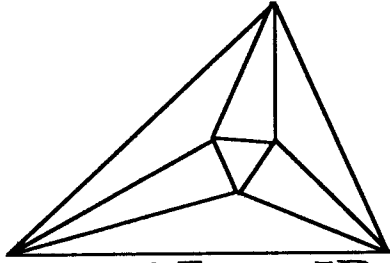
Individual Event

2007-08 Event 5C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

- _____ 1. How many integers between 1000 and 9999 have distinct digits?
- _____ 2. The value of \$500 ten years after having been invested at an annual interest rate of 4% is $\$500(1.04)^{10}$. If the only calculator available has no key for raising to powers, a rough estimate of the value after ten years may be obtained by multiplying 500 by the sum of the first three terms of the expansion of $(1 + .04)^{10}$. What error would be made (in dollars and cents) by using this estimate?
- _____ 3. Distinct integers a and b are chosen independently and at random from the set $\{0, 1, 2, \dots, 2007, 2008\}$. Express as a quotient of relatively prime integers the probability that ab is even.
4. In the game of Yahtzee, five dice are thrown. On such a roll, Pat gets 4, 4, 4, 3, 5. The rules allow her two more chances to roll any subset of the five dice to try to achieve a goal. Hoping for four of a kind (four 4's in this case) or five of a kind (called a Yahtzee), she decides to save the first three 4's, and to roll the other two again. Her strategy is
- if she gets 4, 4, she will have a Yahtzee, and she will quit.
 - If she gets just one 4, she will save it and roll the one other die for her third and final roll.
 - if she gets no 4, she will roll the two non-4's for her third and final roll.
- _____ (a) What is the probability that Pat will wind up with four of a kind?
- _____ (b) What is the probability that Pat will wind up with a Yahtzee?

Name _____ Team _____



Minnesota State High School Mathematics League

Individual Event

2007-08 Event 5D

Questions in this event are written, with permission, as variations of problems from the 2007 AMC-12 Exam. Review of this exam is excellent preparation, not only for this event, but for the 2008 AMC-12, which we strongly encourage you to take.

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. John Q. Public inherited some money. He first gave a tithe (one tenth) of this money to his church. No tax is paid on such a donation. He then paid a 20% federal tax on what he had left, and finally, he paid the state tax of 10% of what was left after paying the federal tax. Taxes paid to the federal and state governments came to \$13,230. How much did he inherit?

(a) _____ 2. For each positive integer n , $S(n)$ is defined to be the sum of the digits of n . Find $n + S(n) + S(S(n))$ for

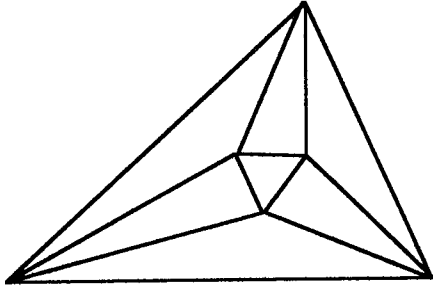
(b) _____ (a) $n = 1978$

(b) $n = 1980$

_____ 3. The set $\{4,7,12,17\}$ is augmented by a fifth positive integer n , not equal to any of the other four. The median of the resulting set is equal to its mean. What is the sum of all possible values of n ?

A(_____) 4. $\triangle ABC$ has an area of 70. $B = (3,4)$ and $C = (7,2)$. Find the point $A(r,s)$ such that $r \geq 0$, $s \geq 0$, and $r + s$ is as large as possible.

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2007-08 Meet 5

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

_____ 1. How many positive integers are composed of four distinct digits such that one digit is the average of the other three?

_____ 2. In $\triangle ABC$, $\angle B = 90^\circ$ and $\sin A + \sin C = \frac{7}{5}$. Find to the nearest tenth of a degree the largest acute angle in $\triangle ABC$.

_____ 3. In the right $\triangle ABC$ (Figure 3), I is the center of the inscribed circle (the incenter). A line through I parallel to the hypotenuse intersects BC at D and AC at E , making $DI = 3$, $IE = 4$. Find the area of $\triangle ABC$.

_____ 4. $\triangle ABC$ is an arbitrary scalene triangle having medians that intersect at M (Figure 4). Find the area of quadrilateral $CEMD$ in terms of $a = \text{Area}(\triangle ABC)$.

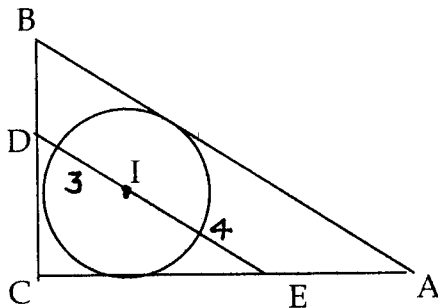


Figure 3

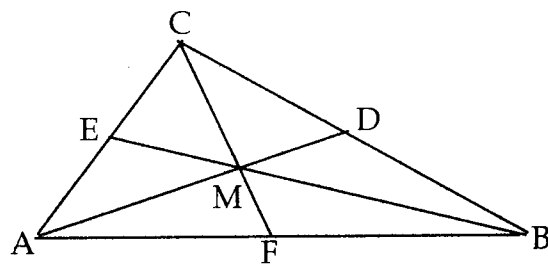
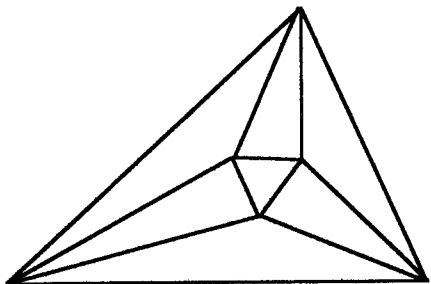


Figure 4

Team _____



Minnesota State High School Mathematics League

Team Event

2007-08 Meet 5

CONTINUED

5. On an $n \times n$ "checkerboard" of boxes, the integers from 1 through n^2 are entered in successive boxes, with the first n integers placed consecutively in row one, the next n integers placed in row two, etc. (Figure 5 shows the checkerboard for $n = 3$.) Suppose we choose n of these integers in such a way that no two of them are in the same row or the same column. Independent of how this choice is made, it turns out that the sum is dependent only on the value of n . In fact, this sum is expressible as a polynomial in n . What is this polynomial?

$p(n) =$ _____

6. In the game of Yahtzee, five dice are thrown. On such a roll, Mary gets 2, 3, 4, 3, 6. The rules allow her two more chances to roll any subset of the five dice to try to achieve a goal. Hoping for a large straight (a sequence of five consecutive integers) or at least a small straight (a sequence of four consecutive integers), she decides to save the first three dice, and to roll the other two again. Her strategy is
- if she gets 1, 5 or 5, 6, she will have a large straight, and she will quit.
 - If she gets just one die that will extend her sequence of consecutive integers (that is a 1 or a 5), she will save it and roll the one other die for her third and final roll.
 - if she gets neither a 1 nor a 5, she will roll both dice again for her third and final roll.

_____ (a) What is the probability that Mary will wind up with a large straight?

_____ (b) What is the probability that Mary will wind up with a small straight?

1	2	3
4	5	6
7	8	9

Figure 5

Team _____