

Minnesota State High School Mathematics League

Individual Event

2007-08 Event 1A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

1. If m , n , and p are distinct prime integers, what least common denominator should _____ be used to add

$$\frac{1}{mn} + \frac{1}{m^2n} + \frac{1}{np} ?$$

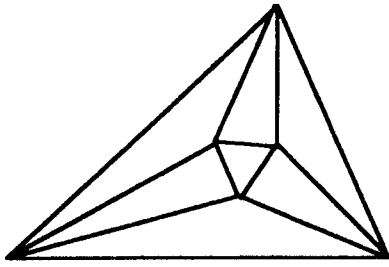
- _____ 2. Express $.666\dots + \frac{1}{1.333\dots}$ as the quotient of two relatively prime integers.

- _____ 3. My new pedometer, when strapped to my ankle, counts the number of steps I take, and then reports the miles I have walked by multiplying the number of steps by the length of a step – which I must enter. The length of step is to be entered as a decimal $m.n$ where m is in feet, n in inches. I first entered 3.1 for a step of 37 inches, but that was evidently incorrect, because the pedometer recorded as 2.4 miles a distance known to be just 2 miles. What length of step $m.n$ (remember that, for example, 2.9 means 2 feet, 9 inches) should I enter to get the best approximation to the correct distance of 2 miles? (There are 5280 feet to a mile.)

- $x_1 =$ _____ 4. Find positive integers x_1 and x_2 , $x_1 < x_2$ such that $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_1 \cdot x_2} = 1$

 $x_2 =$ _____

Name _____ Team _____



Minnesota State High School Mathematics League Individual Event

2007-08 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. Figure 1 shows an equilateral $\triangle AEF$ inscribed in a square $ABCD$. Find the measure in degrees of $\angle AEB$.

_____ 2. Figure 2 shows an equilateral $\triangle BGF$ inscribed in a regular pentagon $ABCDE$. Find the measure in degrees of $\angle DGF$.

_____ 3. In an isosceles $\triangle ABC$, $m(\angle B) = 7m(\angle A)$. Find two possible values for the measure of $\angle C$.

_____ 4. In $\triangle ABC$ (Figure 4), $BE = BF$, $CD = CF$, and $m(\angle A) = 68^\circ$. Find $m(\angle EFD)$.

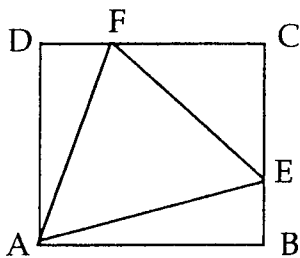


Figure 1

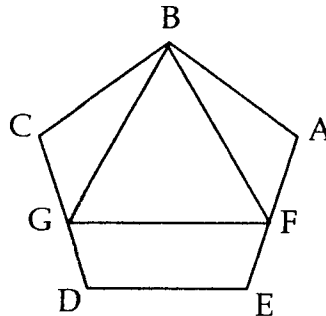


Figure 2

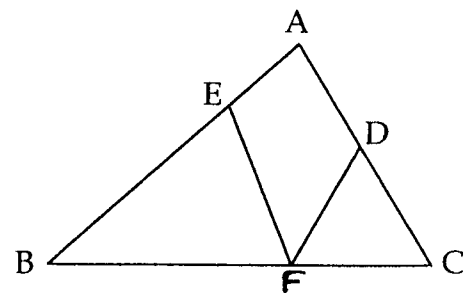
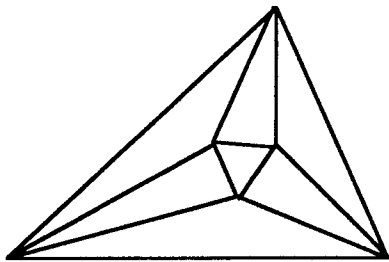


Figure 4

Name _____ Team _____



Minnesota State High School Mathematics League Individual Event

2007-08 Event 1C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. As point $P(x,y)$ moves through the second quadrant following the path of a circle of radius five, OP makes an angle of θ with the positive x -axis (Figure 1). Express $\cos\theta$ in terms of x .

_____ 2. As point $P(x,y)$ moves through the second quadrant following the path of a circle of radius five, OP makes an angle of θ with the positive x -axis (Figure 1). Express $\cos\theta$ in terms of y .

_____ 3. Suppose that in Figure 1, the line from $A(5,0)$ to P has length $5\sqrt{3}$. What is the measure of θ to the nearest degree?

_____ 4. Figure 4A comes from Peter Apianus, *Quadrans Astronomicus* (1532). It depicts two observers trying to determine the height of a tower. The information is shown more clearly in Figure 4B where $DB = 246$, $\angle ADC = 50^\circ$, and $\angle DBC = 25^\circ$. What is the height AC , correct to three places to the right of the decimal?

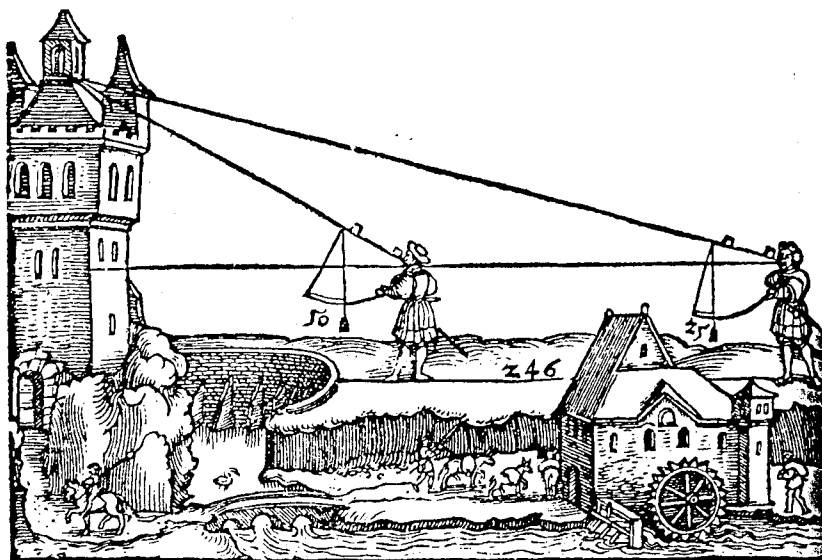


Figure 4A

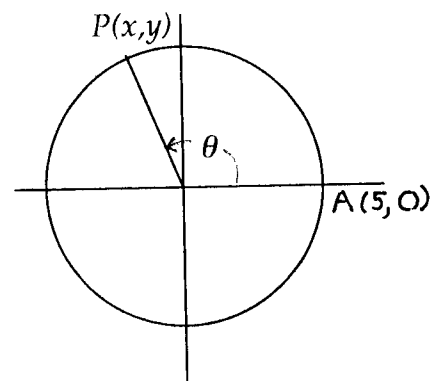


Figure 1

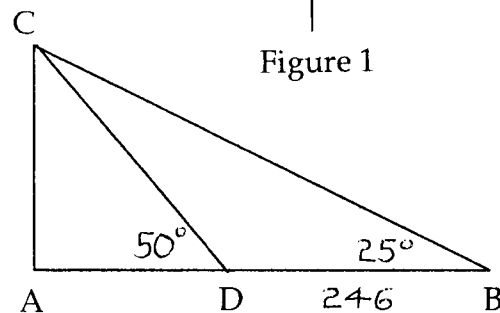
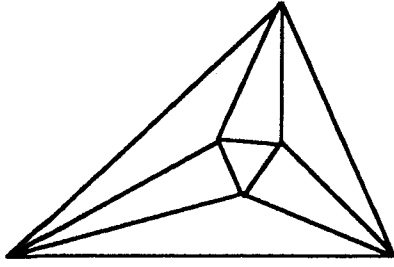


Figure 4B

Name _____ Team _____



Minnesota State High School Mathematics League Individual Event

2007-08 Event 1D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. Find both roots of $(x+1)^2 = -4$.

_____ 2. The polynomial function $f(x)$ has exactly three roots at $x = 1$, $x = -\frac{4}{3}$, and $x = \frac{3}{2}$.

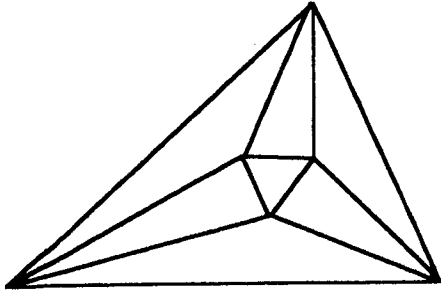
$f(0) = -24$. Find $f(-1)$.

_____ 3. A second degree polynomial function $g(x)$ passes through the points

$\left(\frac{3}{2}, 2\right)$, $(2, 3)$, and $\left(\frac{5}{2}, 1\right)$. Find $g(3)$.

_____ 4. The polynomial $P(x)$ has integer coefficients, and leaves a remainder of -3 when divided by $(x-2)$. The remainder is 17 when $P(x)$ is divided by $(x+3)$. What is the remainder when $P(x)$ is divided by $(x-2)(x+3)$?

Name _____ Team _____



Minnesota State High School Mathematics League

Team Event

2007-08 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

NO CALCULATORS IN THIS EVENT

1. Express using base nine the integer which is written 54321 using base six.

 $x_1 =$

 $x_2 =$

 $x_3 =$

2. Find positive integers $x_1, x_2,$ and $x_3,$ $x_1 < x_2 < x_3$ such that $\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_1 x_2 x_3} = 1.$

3. Figure 3 shows a circle of radius 1 in which BD is tangent to the circle at $C,$ and $AC \perp OB.$ All six trigonometric functions of $\theta = \angle BOC$ can be expressed using a line segment shown on the figure. For example, $\sin \theta = AC$ and $\cos \theta = OA.$ What line segments represent the other four functions? (Your answer must be a line segment shown in the figure; i.e. $\csc \theta = \frac{1}{AC}$ is not allowed.)

$\csc \theta =$ _____ $\sec \theta =$ _____ $\tan \theta =$ _____ $\cot \theta =$ _____

4. The square $ABCD$ (Figure 4) has sides of length 4. E is the midpoint of $CD,$ FG is the perpendicular bisector of $AE,$ meeting it at $H.$ Give the length of GH in exact form.

5. K is a positive two digit number. When its digits are reversed to form the two digit number $L, L \neq K,$ then $K^2 - L^2$ is a perfect square? What is that perfect square?

 $a =$

 $b =$

 $c =$

6. Each zero of $f(x) = ax^3 + bx^2 + cx + 7$ is one more than the reciprocal of a zero of $g(x) = x^3 + x^2 - 5x + 2.$ Determine $a, b,$ and $c.$

Team _____

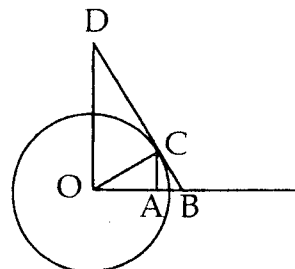


Figure 3

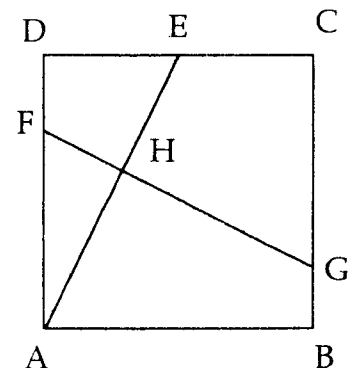


Figure 4