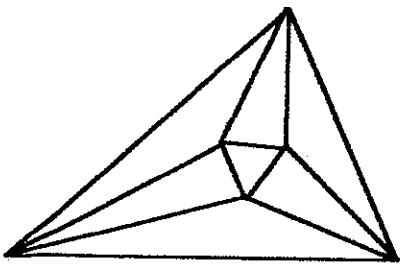


Minnesota State High School Mathematics League

Individual Event

2006-07 Event 1A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. Express as a single integer the least common multiple of the set { 52, 56, 70 }.

_____ 2. Express $\dfrac{\frac{5}{63}+\frac{3}{35}}{\frac{7}{45}+\frac{5}{18}}$ as the quotient of two relatively prime integers.

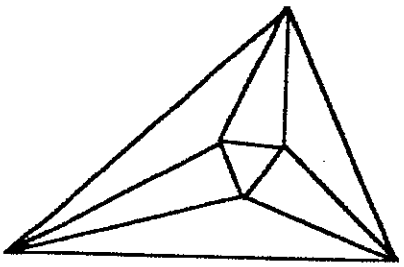
_____ 3. Find the smallest positive integer k such that $\dfrac{7}{39}+\dfrac{k}{117}=\left(\dfrac{a}{b}\right)^2$ where a and b are relatively prime positive integers.

d = _____

r = _____ 4. If d is the greatest common divisor of 399 and 959, then it is possible to find integers r and s so that $d = 399r + 959s$. Find d, r, and s.

s = _____

Name_____Team_____



Minnesota State High School Mathematics League

Individual Event

2006-07 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____1. A straight line intersects the x-axis at A, and the y-axis at B as shown in Figure 1, making $\angle ABO = 70°$. What is the measure of the supplement of $\angle OAB$?

_____2. Referring to Figure 1 and the information given for Problem 1, suppose C is chosen between A and B so that $OC = BC$. What will be the measure of $\angle OCA$?

_____3. In right ΔABC, let D be the mid-point of the hypotenuse BC, and let α be the measure of $\angle BCA$. In terms of α, what is the measure of $\angle ADB$?

_____4. Isosceles ΔABC has its vertex at $\angle A = 30°$ (Figure 3). A trisector of $\angle A$ and a trisector of $\angle B$ meet at R. A trisector of $\angle B$ and a trisector of $\angle C$ meet at S. What is the measure of $\angle BSR$?

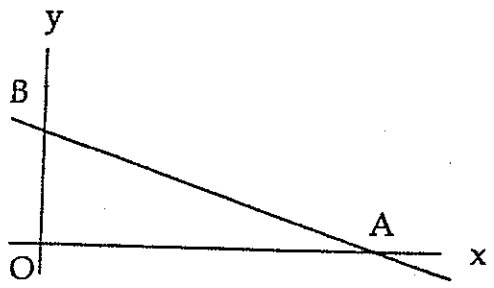


Figure 1

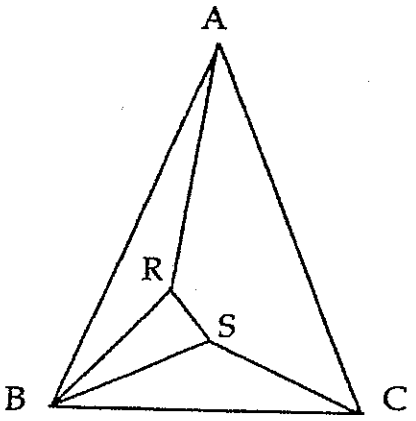
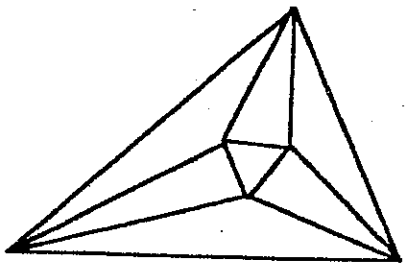


Figure 4

Name_____Team_____



Minnesota State High School Mathematics League

Individual Event

2006-07 Event 1C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. An angle of 195° has a radian measure of $r\pi$ where r is a rational number. What is r?

_____ 2. The smallest acute angle of a right triangle has a sine of 0.4. In exact terms (not a decimal), what is the sine of the largest acute angle?

3. Given that $\cos\alpha > \cos\beta > \frac{1}{\sqrt{2}}$, consider the following three statements.

(a) $\alpha < \beta$ (b) $\alpha > \beta$ (c) $|\alpha| < |\beta|$

Answer each of the two questions below with as many of a, b, and c as seem correct, or answer *none*.

_____Which statements must be true?

_____Which statements must be false?

_____ 4. Figure 4 shows ΔABC with $AC = 4$, $BC = 2$, and a perpendicular dropped from C to D on AB so that $AD = 3DB$. What is the length of AB?

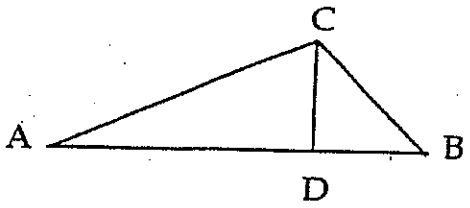
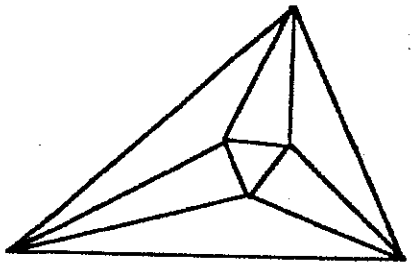


Figure 4

Name_____Team_____



Minnesota State High School Mathematics League

Individual Event

2006-07 Event 1D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

No Calculators in this Event

_____ 1. Find the roots of $6x^2 - 13x + 6 = 0$.

_____

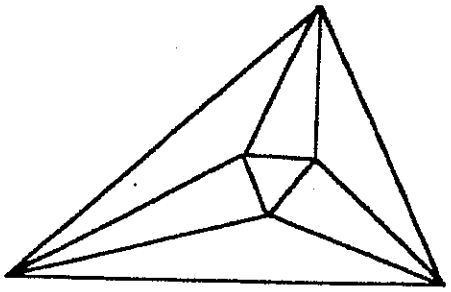
2. Write in descending powers of x the equation of a minimal degree polynomial with integer coefficients having $1 - i$ and $\frac{1}{2}$ as roots.

_____

3. Write the equation of the horizontal line that will be tangent to the graph of
_____ $x^2 - 6x + 2y + 13 = 0$.

_____ 4. If p, q, and r are the roots of $x^3 - x^2 + x - 2 = 0$, what is the value of $p^3 + q^3 + r^3$?

Name_____ Team_____



Minnesota State High School Mathematics League

Team Event

2006-07 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

_____ 1. Two mirrors AB and AC are set at 8° as in Figure 1. A light source is reflected at R_1 where the angle of incidence equals the angle of reflection as indicated. It is then reflected in a similar fashion at R_2, R_3, etc., until, on the nth reflection, it strikes one of the mirrors at a right angle, and then it retraces its path back to C. What is the largest possible value of n?

_____ 2. A newspaper reports that a wall to be built between two warring factions in a city will cost $2 million per kilometer. Using the fact that a kilometer is .62 miles, how much, to the nearest $100,000, will the wall cost per mile?

_____ 3. The right ΔADE in Figure 3 has a side of length 1 opposite the 30° angle at A. From E, lines are drawn to B and C on AD making $\angle EBD = 45°$ and $\angle ECD = 60°$. If a line perpendicular to AD erected at C intersects BE at H, how long (exact form) is HF?

_____ 4. Figure 4 shows ΔABC with $AC = 4$, $BC = 2$, and a perpendicular dropped from C to D on AB so that $AD = 3DB$. To the nearest tenth of a degree, what is the measure of $\angle ACB$?

_____ 5. The graph of $y = \dfrac{x^2 - x - 4}{2(x-3)}$ has a vertical asymptote and an asymptote skew to the x-axis. Find the area enclosed by the two asymptotes and the x-axis.

6. For what choices of k will the graphs of $y = k$ and $y = 2x^3 - 7x^2 - 12x + 6$ have exactly two distinct points of intersection?

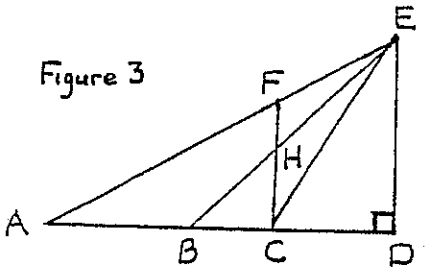


Figure 3

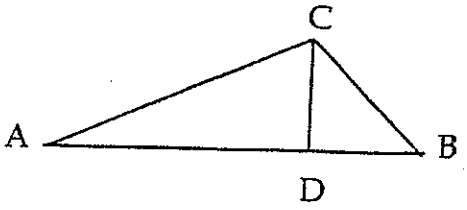


Figure 4

Team_____

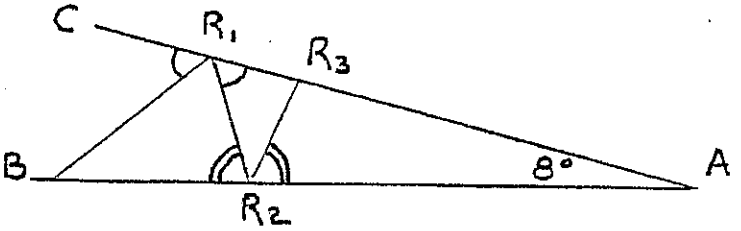


Figure 1