Minnesota State High School Mathematics League

Individual Event

2005-06 Event 1A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

________ 1. Express as the quotient of two relatively prime integers the expression

$$\left[\frac{1}{12}+\frac{1}{15}+\frac{1}{20}\right]^{-1}$$

________ 2. The shirt I was interested in was on a table with a sign, "Take 25% of the marked price." I was about to buy it when the clerk told me that the next day there was to be a sale that would advertise, "Take 1/3 off the already reduced price." If the shirt was originally marked \$37.80, how much will it cost the next day?

________ 3. Find the least common multiple of 108, 84, and 147.

4. The integer $N = 10100$ is expressed using base $b > 1$. Express N as a product of two integers, expressed as polynomials in b, that are both greater than 1.

____

Name________________________________ Team__________________________

Minnesota State High School Mathematics League
Individual Event

2005-06 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

______ 1. Two regular hexagons $ABCDEF$ and $DGHIJK$ (Figure 1) are positioned so that $A, D,$ and I are collinear. If $KD = 2DE$, find the measure in degrees of $\angle KED$.

______ 2. In isosceles ΔABC (Figure 2), the trisectors of the angle at A meet the bisectors of the angles at B and C in points D and E. If $m(\angle BAC) = 36°$, what is $m(\angle BDE)$?

______ 3. In Figure 1, extend AF and IJ to meet at M. Given that $AF = a$, what (in terms of a) is the length of MK?

______ 4. Again refer to Figure 2, but this time do not assume that $\angle A$ is trisected, or that the base angles are bisected. Rather, assume that ΔABC is equilateral, that ΔADE is isosceles with a base DE of length 1, and that both ΔADB and ΔAEC are isosceles. If $AB = 5$, how long is AD?

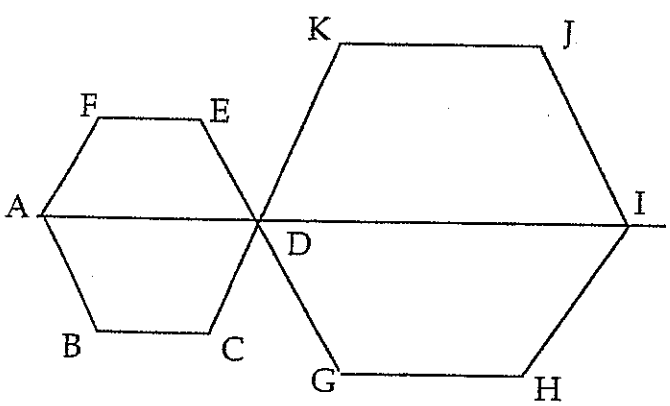

Figure 1

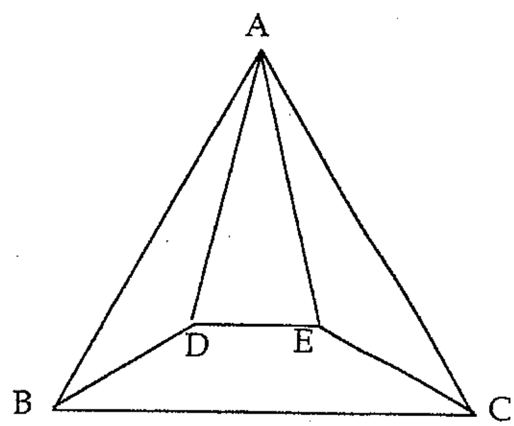

Figure 2

Name______________________________ Team______________________

Minnesota State High School Mathematics League

Individual Event

2005-06 Event 1C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

______ 1. Figure 1 shows the graph of $y = a\sin bx$. What is b?

______ 2. Again refer to the graph $y = a\sin bx$ in Figure 1. Given that the units on the x and y axes are the same, and that a is an integer, what is the value of a?

______ 3. In isosceles ΔABC (Figure 3), the trisectors of the angle at A meet the bisectors of the angles at B and C in points D and E. A perpendicular is dropped from D to a point F on AB. If $m(\angle BAC) = 36°$ and $AB = 5$, how long is AF?

______ 4. Refer again to Figure 3 and the information given in Problem 3. How long is DE?

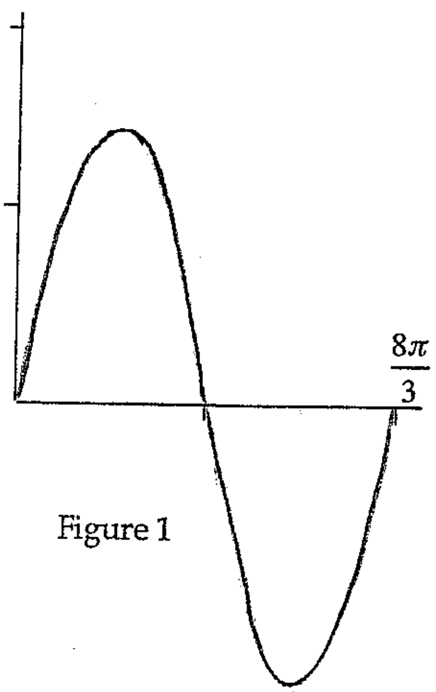

Figure 1

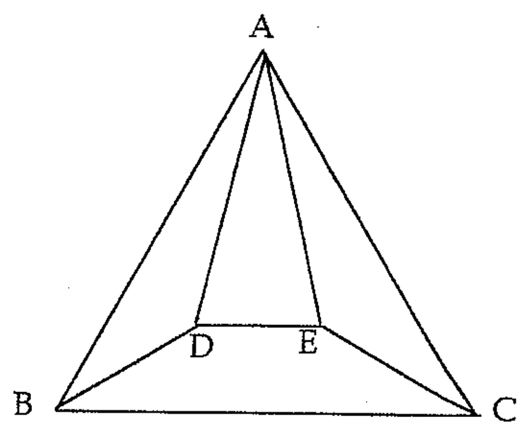

Figure 3

Name______________________________ Team______________________

Minnesota State High School Mathematics League

Individual Event

2005-06 Event 1D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

______ 1. Find the roots of $(2x+1)^2 = \frac{3}{4}$

______

______ 2. What will be the remainder if $2x^5 - 3x^4 - 4x^3 - 5x^2 - 6x - 7$ is divided by $x-3$?

3. Find the coordinates (both of them) of the lowest point on the graph of

______ $9x^2 + 24x - 72y - 164 = 0$.

4. The coefficients of $z^3 + az^2 + bz + c = 0$ are all real numbers. The three roots z_1, z_2, and z_3 of the equation satisfy

$$z_1z_2z_3 = 1 \qquad \text{and} \qquad z_1 + z_2 + z_3 = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$$

For what choices of a will there be some non-real complex roots?

______

Name______________________ Team______________________

Minnesota State High School Mathematics League

Team Event

2005-06 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

1. Express $\left[\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}\right]^{-1}$ as a single quotient.

2. The integer $M = 100011$ is expressed using base $b > 1$. Express M as a product of two integers, expressed as polynomials in b, that are both greater than 1.

3. Find to the nearest tenth of a degree all angles θ, $0 \le \theta < 360$, that satisfy
$$15\tan^2\theta - \tan\theta\sec\theta - 6\sec^2\theta = 0$$

4. In ΔABC, $\angle A = 40°$, $\angle B = 25°$, and $AC = 3$. What is the area of ΔABC?

5. For distinct real numbers a, b, and c, division of the polynomial $p(x)$ by $x-a$ leaves a remainder of a; division of the polynomial $p(x)$ by $x-b$ leaves a remainder of b; division of the polynomial $p(x)$ by $x-c$ leaves a remainder of c. What, in simplest form, is the remainder when $p(x)$ is divided by $(x-a)(x-b)(x-c)$? Hint: rephrase the question. Let $p(x) = (x-a)(x-b)(x-c)q(x) + r(x)$. Find $r(x)$ in simplest form.

6. Given: $x+y+z=0$; $\quad x^2+y^2+z^2=36$; $\quad x^3+y^3+z^3=105$
One of the unknowns, say x, (by symmetry, it could be any one of them) is real; the other two are complex. Find x.

Team ______________________________