# Minnesota State High School Mathematics League

## Individual Event

## 2004-05 Event 1A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

*Write the expressions in problems 1 and 2 as the quotient of two relatively prime integers.*

_____ 1. $.272727\ldots = $

_____ 2. $\dfrac{\dfrac{1}{6}+\dfrac{3}{4}}{\dfrac{4}{9}+\dfrac{2}{3}} = $

_____ 3. Carrie A. Handful found on line 14 of her 2003 Minnesota Income Tax that her taxable income was $243,812. She then read, "If line 14 is over $110,390, enter on line 15 $7310.24 + 7.85% of the amount over $110,390." How much should she enter on line 15?

_____ 4. The set of three positive integers $\{12, 45, m\}$ has a greatest common divisor $d > 1$ and a least common multiple of 2520. What is the sum of the possible values for the integer $m$?

Name_____Team_____
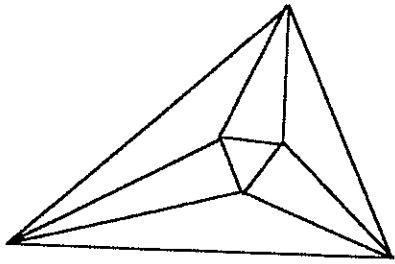
# Minnesota State High School Mathematics League

## Individual Event

### 2004-05 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. Parallel lines pass through the vertices $A$ and $C$ of square $ABCD$ as shown in Figure 1. If $\angle ECD = 64°$, what is the measure of $\angle FAD$?

_____ 2. Isosceles $\triangle ABC$ has as its apex $\angle A = 90°$. $BD$ is drawn so that $\angle CBD = \frac{1}{3}\angle CBA$, and it is extended to meet a line erected at $C$, perpendicular to $AC$ at $E$ (Figure 2). What is the measure of $\angle BEC$?

_____ 3. In $\triangle ABC$ with $AC < BC$, let the measure of the angles at $A$ and $B$ be $\alpha$ and $\beta$ respectively. Extend $CA$ to $D$ so that $CD = CB$. Express the measure of $\angle CDB$ in terms of $\alpha$ and $\beta$.

_____ 4. In isosceles $\triangle ABC$ with base $BC$ and $D$ as the midpoint of leg $AC$, $BC = BD = DA$. What is the measure of $\angle BAC$?
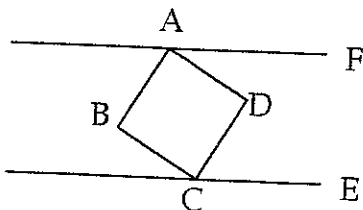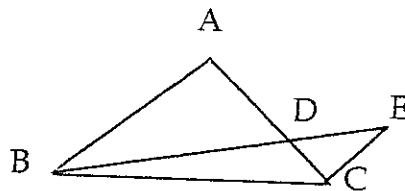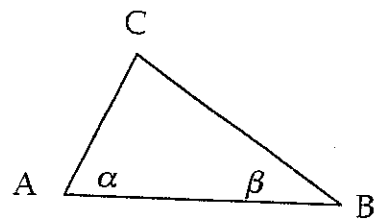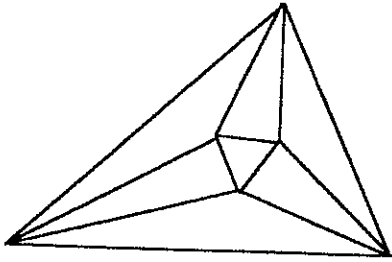
Figure 1

Figure 2

Figure 3

Name _____ Team _____

# Minnesota State High School Mathematics League
## Individual Event

## 2004-05 Event 1C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. Express in exact, simplified form the value of $\cos\dfrac{5\pi}{6} + \tan\dfrac{\pi}{3}$.

_____ 2. The line from the origin $O$ to the point $A(-4,-2)$ forms with the positive $x$-axis an angle $\alpha$ that is between $\pi$ and $\dfrac{3\pi}{2}$. If $\csc\theta = \sqrt{5}, 0 < \theta < \dfrac{\pi}{2}$, express $\alpha$ as a radian expression involving $\theta$.

3. Two tangents to a circle, each of length 12, intersect to form an angle of 60°. If the tangents meet the circle at points $A$ and $B$, the length of the major arc $AB$ is $k\pi$. Give the exact, rationalized value of $k$.

4. Hipparchus (190-120 B.C.) estimated the distance from the surface of the earth to the moon by the following method, described here using miles as the unit of measurement. Using an eclipse of the moon as the signal for obtaining simultaneous observations, it was found that when observers were separated by 6220 miles (measure in the usual way along the surface of the earth), observer A could see the moon M just over the horizon while observer B saw it directly overhead (Figure 1). Estimates of the radius of the earth at the time were 4000 miles. Using these numbers, what is BM to the nearest mile?
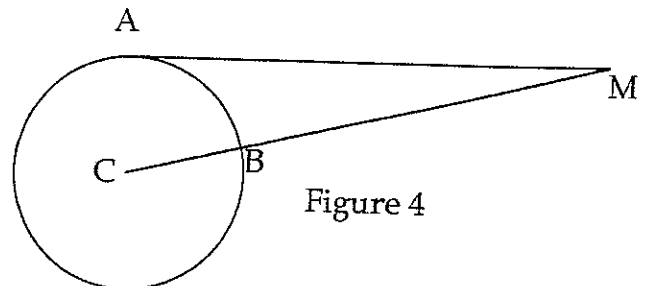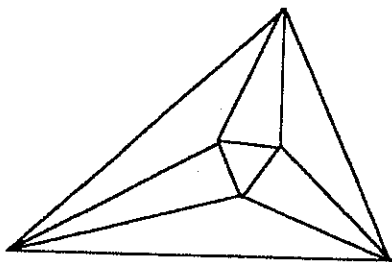


Figure 4

Name_____Team_____

# Minnesota State High School
## Mathematics League
### Individual Event

## 2004-05 Event 1D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

*No calculators on this event.*

_____ 1. What is the product of the roots of $2x^2 - 3x + 2 = 0$?

_____ 2. Find $b$ so that the equation $4x^2 + bx + 9 = 0$ has one rational (double) root. (contributed at the 2003 coaches conference)

_____ 3. Given that $-2$ is one root of $6x^3 - 5x^2 - 29x + 10 = 0$, find the other two roots

_____ 4. The polynomial $3x^3 - 13x^2 + ax + b = 0$, $a$ and $b$ real numbers, has $2 - i$ as one of its roots. Where does the graph of $y = 3x^3 - 13x^2 + ax + b$ cross the $x$-axis?

Name_____Team_____
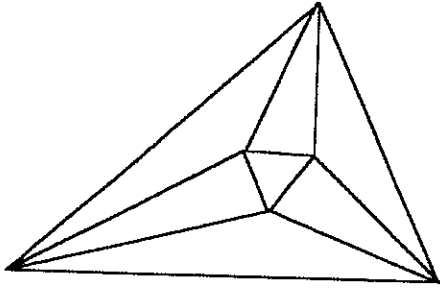
# Minnesota State High School Mathematics League

## Team Event

### 2004-05 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

_____ 1. Isosceles $\triangle ABC$ has as its apex $\angle A = 90°$. $BD$ is drawn so that $\angle CBD = \frac{1}{3}\angle CBA$.

Line $CE$ is drawn perpendicular to $AC$ and meets the extension of $BD$ at $E$ (Figure 1). If the legs of the triangle have length 1, what is the length of $CE$?

_____ 2. Isosceles $\triangle ABC$ has as its apex $\angle A = 90°$. Adjacent trisectors of its angles meet to form $\triangle DEF$ as shown in Figure 2. What (in degrees) is the measure of $\angle DEC$?

_____ 3. For what value(s) of $x$ is $f(x) = |-3 - x| + |1 - x|$ a minimum?

_____ 4. An observer on a boat B traveling due east notices a light 20° to the northeast. After traveling 1 mile, the light is 40° to the northeast (Figure 4). Find the distance $x$ in miles that the boat will have to traveled when the light L is due north.

b=_____
k=_____ 5. The equation $9x^2 + 12x = 32$ can be written in the form $(x + b)^2 = k^2$. Find (i) $b$; (ii) $k$.

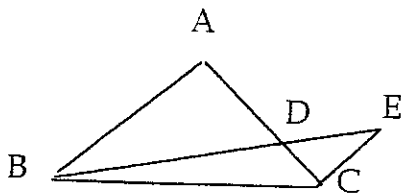_____ 6. Give a numeric value for the continued fraction $\cfrac{3}{2 + \cfrac{3}{2 + \cfrac{3}{2 + \cfrac{3}{2 + \cdots}}}}$

Figure 1

Figure 2
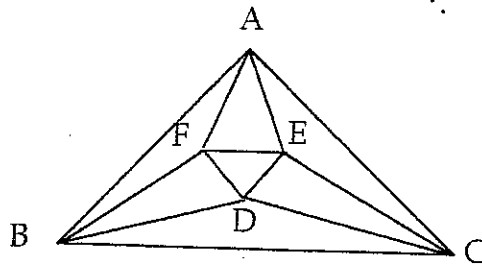
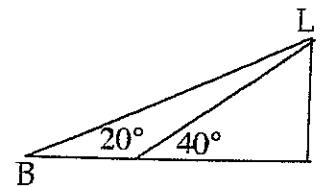Figure 4

Team_____