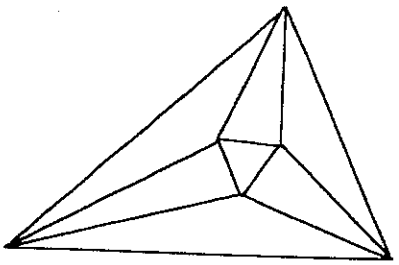


Minnesota State High School Mathematics League

Individual Event

2003-04 Event 1A

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. The decimal 1.545454 . . . can be expressed as $\frac{17}{11}$, the quotient of two relatively prime integers. Express 1.272727. . . as the quotient of two relatively prime integers.

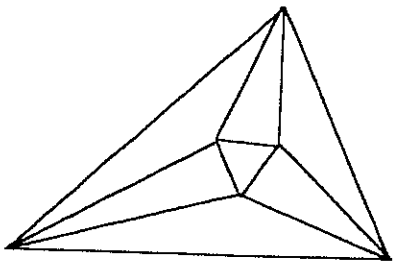
_____ 2. When a group of Mathleaguers went to a Twins game last summer, we got with our tickets a brochure saying that with each ticket we could buy a coupon for \$3.50 that would be good for a hot dog and a soft drink. This, it said, would be a saving of more than 30% off the regular price. The regular price for a hot dog and soft drink was \$5.75. What (to the nearest tenth of a percent) was the actual saving?

3. Let $0 < a < \frac{1}{4} < \frac{3}{4} < b < 1$. Use a similar string of inequalities to order from smallest to largest $\sqrt{a}$, a^2, $\frac{1}{a}$, $\sqrt{b}$, b^2, $\frac{1}{b}$,

_____

4. The set of three positive integers $\{15, 25, k\}$ has a greatest common divisor of 5 and
_____ a least common multiple of 450. What is the sum of the possible values for the integer k?

Name_____ Team_____



Minnesota State High School Mathematics League

Individual Event

2003-04 Event 1B

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. Figure 1 shows an equilateral ΔRST inscribed in a regular pentagon $ABCDE$ in such a way that R is the midpoint of $\overline{CD}$ and $\overline{ST} \parallel \overline{CD}$. What is the measure of $\angle ATS$?

_____ 2. In Figure 1, what is the measure of $\angle ESR$?

_____ 3. In an isosceles ΔABC with $AC=BC$, let $\overline{CD}$ and $\overline{CE}$ be the angle trisectors of $\angle C$, and let $\overline{AF}$ and $\overline{AG}$ be the angle trisectors of $\angle A$ (Figure 3). Let $\overline{AF}$ intersect $\overline{CD}$ at K and $\overline{CE}$ at L. If $\angle A = 54°$, what will be the measure of $\angle KLE$?

_____ 4. If in Figure 3, we also draw the angle trisectors of $\angle B$ and let R, S, and T be the points where pairs from adjacent vertices intersect (as in the logo at the top of the page), what will be the measure of $\angle ART$?

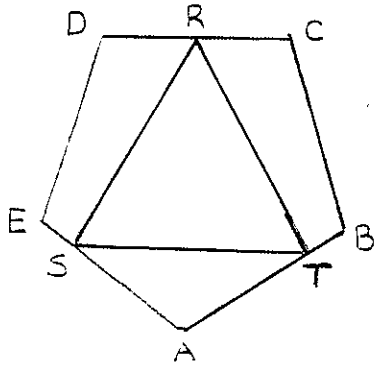


Figure 1

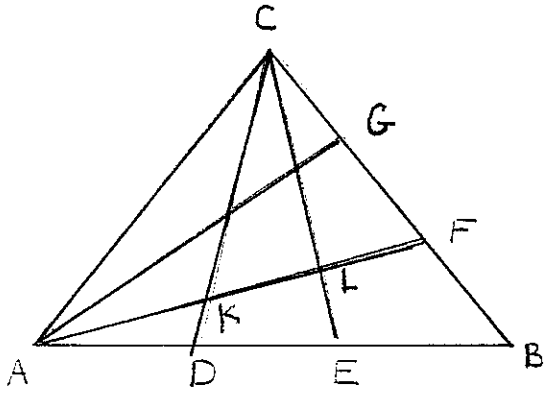
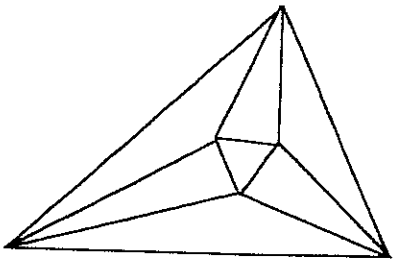


Figure 3

Name_____ Team_____



Minnesota State High School Mathematics League

Individual Event

2003-04 Event 1C

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

_____ 1. What is the sine of the second quadrant angle α shown in Figure 1?

_____ 2. For the angle α shown in Figure 1, what is $\sin\left(\alpha + \frac{\pi}{2}\right)$?

3. Express $\sqrt{\frac{1-\sin x}{1+\sin x}}$ as the difference of two trigonometric functions (such as $\sin x - \csc x$), given that x is in the second quadrant.

_____

4. Two tangents to a circle, each of length 12, intersect to form an angle of 30°. If the tangents meet the circle at points A and B, what is the length, accurate to three places to the right of the decimal, of the long arc $\overset{\frown}{AB}$?

_____

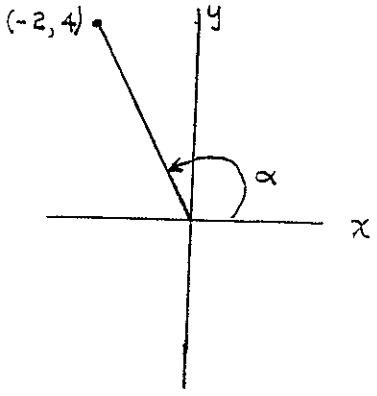
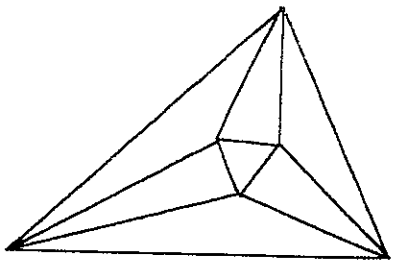


Figure 1

Name_____ Team_____



Minnesota State High School Mathematics League

Individual Event

2003-04 Event 1D

The first question is intended to be a quickie and is worth 1 point. Each of the next three questions is worth 2 points. Place your answer to each question on the line provided. You have 12 minutes for this event.

All questions in this event refer to the polynomial $p(x) = 2x^2 + 2x + 3$

_____ 1. What is the product of the roots of $p(x) = 0$?

2. The graph of $y = p(x)$ intersects the graph of $y = 5 - x$ in two points A and B. Give the coordinates (both of them) of both points.

A _____

B _____

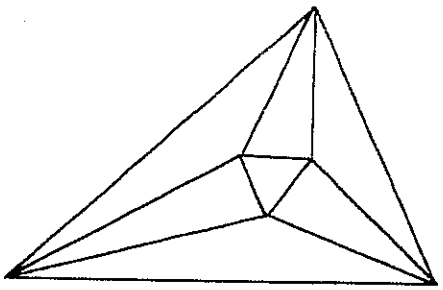
3. Write $y = p(x)$ in the form $y - k = 4a(x - h)^2$.

_____

4. The equation $6x^4 + 10x^3 + 15x^2 + 8x + 3 = 0$ has two of the same roots as the equation $p(x) = 0$. For what second degree polynomial $r(x)$ does $r(x) = 0$ have roots equal to the other two roots of the given 4th degree polynomial equation?

_____

Name_____ Team_____



Minnesota State High School Mathematics League

Team Event

2003-04 Meet 1

Each question is worth 4 points. Team members may cooperate in any way, but at the end of twenty minutes, one set of answers is to be submitted. Put answers on the lines provided.

_____ 1. The set of four positive integers $\{42, 54, 60, k\}$ has a greatest common divisor of 6 and a least common multiple of 7560. What is the sum of the possible values for the integer k?

2. Figure 2 shows a circle of radius r with a central angle θ, $0 \le \theta \le \frac{\pi}{2}$. The inequalities $Area(\Delta OBD) < Area(\text{sec}tor\ OBD) < Area(\Delta OBC)$ can be used to show $f(\theta) < \theta < g(\theta)$ where $f(\theta)$ and $g(\theta)$ represent trigonometric functions. Write the inequality.

_____

3. Describe a good calculator window in which one can see a continuous part of the graph of $y = P(x) = 30x^3 - 121x^2 + 162x - 72$ that shows all the roots of $P(x) = 0$.

xmin= _____ xmax= _____ ymin= _____ ymax= _____

_____ 4. Give a numeric value for the continued fraction $\cfrac{6}{1+\cfrac{6}{1+\cfrac{6}{1+\cfrac{6}{1+\ddots}}}}$

5. Figure 5 shows the graph of a parabola $y = x^2 + bx + c$ having its lowest point at _____ (m,n), $n < 0$. Express the roots of $x^2 + bx + c = 0$ in terms of m and n.

6. Let $f(x) = 3x^2 - 2(a+b+c)x + (ab+ac+bc)$. Then $f\left(\frac{a+b}{2}\right)$ can be expressed as a rational number times the square of a term involving a and b. Do so.

_____

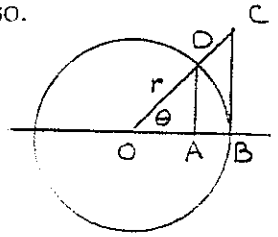


Figure 2

Team_____